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A Bayesian-reliability based multi-objective optimization for tolerance design of mechanical assemblies

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ABSTRACT

Tolerances significantly affect the assemblability of components, the product's performance, and manufacturing cost in mechanical assemblies. Despite the importance of product reliability assessment, the reliability-based tolerance design of mechanical assemblies has not been previously considered in the literature. In this paper, a novel method based on Bayesian modeling is proposed for the tolerance-reliability analysis and allocation of complex assemblies where the explicit assembly functions are difficult or impossible to extract. To reach this aim, a Bayesian model is developed for tolerance-reliability analysis. Then, a multi-objective optimization formulation is proposed for obtaining the optimum tolerances of components to minimize cost and maximize product performance. Subsequently, Non-dominated Sorting Genetic Algorithm II (NSGA-II) is employed for solving multi-objective optimization. Then, the enhanced TOPSIS is used to find the best optimum tolerances from the optimum Pareto solutions. Using the importance vector concept, a sensitivity analysis approach is used to determine the effects of design variables on the product reliability level and improve assembly reliability to the desired level. Finally, to exhibit the applicability of the proposed method, a transmission planetary gear system is considered, and the obtained results are compared and discussed for verification.

1. Introduction

Due to variations that arise during imperfect manufacturing processes, attaining the theoretical dimensions and consequently, the desired quality and performance may not be possible for a product. In such a condition, tolerance design which is a repeatable process consisting of tolerance analysis and tolerance synthesis steps plays a pivotal role in ensuring the feasibility and quality of mechanical assemblies at a lower cost. In general, the tolerance analysis involves evaluating the accumulation of the component tolerances on assembly dimensions or key characteristics in a mechanical system. The accumulation of component tolerances can affect assembly dimensions and key characteristics of mechanical assembly that should satisfy functional requirements. On the other hand, optimal tolerances should be assigned to individual dimensions considering the assemblability and functionality requirements in the tolerance synthesis stage.

Over the last few decades, several studies have been conducted to develop the mathematical basis for tolerance analysis of mechanical assemblies. The most well-known tolerance analysis methods are reviewed in several studies [1–3]. Chase et. al. [4] presented the direct linearization method for tolerance analysis of mechanical assemblies using small kinetic adjustment of component dimensions. Laperrière and Lafond proposed a kinematics-based method for tolerance analysis and synthesis based on the Jacobian transform concept [5]. In this method, for tolerance modeling, all small displacements of geometric features are considered. This concept has been applied as a useful basis for developing several tolerance analysis techniques in the literature (e.g. [5]). A study [6] proposed the TTRS model using rigid body motion and various concepts. Desrochers et al. [7] developed the unified Jacobian-Torsor model by combining the Jacobian and Torsor models. This method uses the Torsor model and Jacobian matrix to represent tolerances and tolerance propagation respectively. Khodaygan et al. (2010) proposed a feature-based tolerance analysis method that can cover all geometrical and dimensional tolerances that is compatible with GD&T standards [8]. Ziegler and Wartzack (2015) proposed an approach to adopt Sensitivity Analysis methods on current tolerance simulations with an interface module, which bases on level sets of constraint functions for parameters of the simulation model [9]. Khodaygan et al. (2011) developed an uncertainty–accumulation model for tolerance analysis of mechanical assemblies based on fuzzy logic [10]. Also, Khodaygan and Movahhedy

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(2016) introduced a comprehensive method for the tolerance analysis based on the combined fuzzy- small degrees of freedom F-SDOF model [11]. In this method, rules and concepts in GD&T standards have been modeled. In another study, Khodaygan and Ghaderi (2019) developed a tolerance–reliability analysis method for quality control of mechanical assemblies based on Bayesian modeling [12]. In order to consider the parts' form defects in tolerance analysis, Homri et al. [13] developed a Metric Modal Decomposition (MDD) method. By combining the Jacobian and skin model shape approaches, Liu et al. [14] proposed a novel method to model the actual toleranced surfaces. This method uses a Jacobian matrix and a skin model shape for tolerance transformation calculation and toleranced features description, respectively. Tlija et al. [15] presented a new model for the tolerance analysis of CAD assemblies considering the manufacturing defects and deformations. Further, Tlija et al. [16] developed a new method based on difficulty coefficient evaluation (DCE) and Lagrange multiplier (LM) methods to obtain an economical tolerance allocation.

Corrado al. (2018) proposed a variational model for tolerance analysis which can deal with form tolerances and assembly conditions [17]. In another study, Polini and Corrado (2019) implemented the geometric reasoning in the model to simulate the manufacturing process and, then, the assembly sequence [18]. Also, Corrado and Polini (2020) conducted a comparative study on some tolerance analysis methods in the literature to compare the performance of methods [19]. Anwer et al. [20] investigated the fundamentals of the skin model at a conceptual, geometric, and computational level as well. Kong et al. (2020) proposed an approach for the tolerance design and process parameter analysis that can consider performance degradation [21]. Moreover, in this work, a reliability model with temperature-humidity-mechanical stress covariate has been presented.

In literature, several methods of tolerance allocation have been developed based on the minimum manufacturing cost and the optimum performance of mechanical systems [22]. Huang and Shiau (2009) proposed an optimal tolerance allocation model considering the manufacturing cost, quality loss, and the design reliability index [23]. Balamurugan (2017) developed a concurrent tolerance allocation model to consider the degradation effects on the product's quality characteristics [24]. To calculate the manufacturing cost more accurately, Lui et al. (2017) proposed to use different kinds of manufacturing cost functions for various components of the same assembly [25]. Natarajan et al. (2018) developed a bi-objective manufacturing tolerance allocation model for an interchangeable assembly of shaft and hole [26]. Khodaygan (2019) proposed a multi-objective optimization formulation for optimum asymmetric tolerance design [27]. In other work, Khodaygan (2019) developed an optimum tolerance design of compliant assemblies [28]. In this method, error propagation due to the flexibility of components in stages of the assembly process is estimated via the enhanced Method of Influence Coefficients (MIC). Liu et al. [29] proposed a modified quality loss function based on wear regularity. To model the loss function accurately, the change of main quality characteristics over service life was modeled nonlinearly. Then, the service life distribution determined based on the nonlinear wear regularity was used to improve the quality loss function. Hassani et al. proposed a Reliability-Based Robust Design Optimization method in presence of both aleatory and epistemic uncertainties [30]. The proposed formulation was presented as a multi-objective optimization problem. For approximating the mean and the variance of the design function, the univariate dimension reduction method was applied. Vahidi Moghaddam et al. proposed an optimization method for fuzzy problems to consider the uncertainty of nonlinear systems [31,32].

One of the main ways to sure the desired performance of a product is the reliability assessment of the product. Uncertainties that arise in the design process or performance period of the product may affect the feasibility of the design and consequently decrease the product quality. Therefore, the constraints of the tolerance allocation problem can be reformulated in probabilistic forms to deal with existing uncertainties and ensure reliability. The major conclusion of methods that have been reviewed in the introduction section can be classified into the following categories:

- Despite the importance of reliability assessment in the tolerance allocation of product, the tolerance - reliability design of mechanical assemblies has not been previously considered in the literature.
- There is not a tolerance design method based on the experimental observations by Bayesian inference.
- Conventional methods by expanding the assembly function into a Taylor series so that an inaccurate linear assembly function with constant coefficients is obtained.

To overcome the above-mentioned gaps in the literature, in this study, a novel Bayesian-based sequential tolerance allocation and reliability assessment algorithm has been proposed for optimal tolerance synthesis of mechanical assemblies while ensuring the design reliability.

The main novelties of the proposed method can be summarized as follows:

(1) The reliability assessment is a significant task in the tolerance allocation of products. In this paper, unlike previous works in the literature, a reliability-based optimal tolerance design of mechanical assemblies is proposed. In other words, using the obtained optimal tolerances from the proposed method can guarantee the design reliability of the system.
(2) The proposed method can model the explicit assembly function of the complex assembly where it is difficult or impossible to find by existing conventional methods in the literature.
(3) As a new approach, the proposed method takes into account the cost of quality control in addition to the cost of construction.

To approximate the assembly function based on experimental observations and decrease the meta-modeling uncertainties, which is one of the most common kinds of epistemic uncertainty, Bayesian linear regression has been used in the proposed algorithm. The tolerance allocation problem has been formulated as a bi-objective optimization problem contains minimizing total cost (summation of quality loss cost, manufacturing cost, rejection cost, and inspection cost) and minimizing the variation of functional characteristics. To solve the bi-objective tolerance allocation problem, the elitist Non-dominated sorting genetic (NSGA-II) algorithm has been used. Then, an enhanced Shannon's entropy-based TOPSIS algorithm, as a multi-criteria decision tool, has been applied to find the best tolerances from the non-dominated minimum total cost and minimum functional characteristic variation solutions. Finally, the importance vector has been used to shift the tolerances toward the feasible region and increase the design reliability to an acceptable level. In this study, the first-order reliability method (FORM) using gradient-based improved Hasofer-Lind and Rackwitz-Fiesler (iHLRF) searching algorithm has been applied to assess the reliability for the best tolerances. So that using the proposed method improves the product reliability to achieve the desired level under minimum total product cost and maximum quality of the product.

The remainder of this paper is organized as follows; In Section 2, the proposed approach is introduced. Then, the optimum tolerance design of a transmission planetary gear system as a case study is carried out by the proposed method in Section 3. Finally, the paper is finished by the conclusions in Section 4.

2. Proposed method

In this section, the proposed method for multi-objective reliability-based tolerance design based on Bayesian modeling is introduced. The proposed method can be presented in the following steps:

Step 1 Bayesian modeling of design function based on experimental observations
Step 2 Formulating the multi-objective tolerance allocation problem
Step 3 Extracting the non-dominated Pareto front of optimum tolerances for components
Step 4 Selecting the best optimum tolerances from the obtained Pareto front
Step 5 Bayesian reforming optimum tolerances to improve the reliability of the assembly
To a better understanding, the main stages of the proposed method in the following steps are explained in detail:

2.1. Bayesian modeling of design function based on experimental observations

In general, there are two main views in statistics: frequentist or classical statistics and Bayesian statistics [33]. Bayesian model as a more flexible tool to model complex relationships can be an accurate computational technique by taking into account previous information about the phenomenon under investigation where observations are limited. The general expression of design function by Bayesian linear regression model can be written as follows:

$$t_y = \theta_1 t_{x_1} + \theta_2 t_{x_2} + \ldots + \theta_n t_{x_n} + \varepsilon, \tag{1}$$

where t_y and t_{x_i} denote tolerances of the functional variable (y) and design variables (x_i), respectively. Also, θ_i indicates model parameters and ε is a model error. Based on the experimental data, the Bayesian design function model can be expressed as follows:

$$\widetilde{t}_y = \widetilde{T}\widetilde{\theta} + \widetilde{\varepsilon}, \tag{2}$$

where:

$$\widetilde{t}_y = \left[t_{y_1}, t_{y_2}, \ldots, t_{y_m}\right]^T, \widetilde{T} = \begin{bmatrix} t_{x_{11}} & \cdots & t_{x_{1m}} \\ \vdots & \ddots & \vdots \\ t_{x_{n1}} & \cdots & t_{x_{nm}} \end{bmatrix}, \widetilde{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]^T, \widetilde{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m]^T, \tag{3}$$

where n is the number of parameters and m is the number of experiments.

Let $\widetilde{\theta} = [\theta_1, \ldots, \theta_n]^T$ be all of the unknowns of model, which we continue to refer to as parameters, and $\widetilde{t}_y = [t_{y_1}, \ldots, t_{y_m}]^T$ the vector of observed data. Bayesian inference is based on the posterior probability distribution of θ after observing t, which is given by Bayes theorem [33, 34]:

$$\mathscr{P}''(\theta|t) = \frac{l(t|\theta)}{C(t)}\mathscr{P}'(\theta) \tag{4}$$

where $\mathscr{P}'$ and $\mathscr{P}''$ are the prior and posterior probability, respectively. $l(t|\theta)$ is likelihood to indicate the compatibility of evidence t with the given θ. $C(t|\theta)$ is marginal likelihood that is the same for all possible θ being considered two key ingredients: the likelihood function $l(t|\theta)$ and the prior distribution $\mathscr{P}'(\theta)$. The latter represents the probability beliefs for θ held before observing the data t. The normalizing constant ($C(t)$) can be written as follows:

$$C(t) = \int l(t|\theta)\mathscr{P}'(\theta)\mathrm{d}\theta \tag{5}$$

And also, the normalizing constant is the marginal probability of the observed data given the model, that is, the likelihood and the prior.

It is worth to be noted that the proposed model can be applied for non-Gaussian data-set as well; however, according to the literature [35, 36], tolerance data-sets usually follow the Gaussian distribution. In the case of non-Gaussian distribution, due to the property of Bayesian inference, model parameter θ_i have always the Gaussian distribution. However, the computational complexity in the data set with the non-Gaussian probability density functions may be more than the Gaussian probability density function.

2.2. Formulating the multi-objective tolerance allocation problem

In this section, optimum tolerance allocation is formulated in a multi-objective optimization problem.

2.2.1. Formulating objective functions for the optimum tolerance allocation

In this subsection, the total cost and the functional requirement are considered as the main objectives to formulate the tolerance allocation problem.

2.2.1.1. Modeling the total cost function. In general, the optimum tolerance design procedure is a trade-off between performance and production cost. Therefore, for allocating optimum tolerances of components, the main sources of production cost should be modeled in terms of tolerances as the cost function of the multi-objective optimum tolerance procedure.

- *Cost of quality reduction*

To take into account the additional cost due to quality reduction, the concept of the quality loss function can be used. In general, the quality loss function refers to cost due to the reduction of quality loss where the quality variable ($\widetilde{t}_y$) has deviated from the target value ($\widetilde{t}_{y_d}$) [25]. The quality loss function can be expressed in a quadratic form as below:

$$QLC(y) = k\left(\widetilde{t}_y - \widetilde{t}_{y_d}\right)^2, \quad when \quad k = \frac{A}{\Delta^2}, \tag{6}$$

where k indicates loss coefficient, A is the cost increment due to a deviation (Δ) from the target value.

For statistical tolerance analysis, the root sum square (RSS) approach can be applied under two assumptions; design variables are independent under normal distribution [10,37]. Referring to Eq. 1, the Bayesian linear regression model can be rewritten in the RSS-based form as follows:

$$t_y = \left\{\sum_{i=1}^{n}\left(\theta_i t_{x_i}\right)^2\right\}^{1/2}. \tag{7}$$

Substituting Eqs. 7 into 6, quality loss cost can be rewritten as follows:

$$QLC_{transemile} = K\left\{\left\{\sum_{i=1}^{n}\left(\theta_i t_{x_i}\right)^2\right\}^{1/2} - t_{y_d}\right\}^2 \tag{8}$$

- *Manufacturing cost*

In the literature, several cost-tolerance models for tolerance allocation of mechanical assemblies have been introduced [38].

$$C = C_0 + \frac{A}{t^k}, \tag{9}$$

where C_0 is the constant cost of manufacturing setups, A and k denote cost coefficient and cost exponent in the manufacturing a dimension with tolerance t, respectively. Accordingly, the total manufacturing cost of all effective dimensions can be obtained below:

$$MC = \sum_{i=1}^{n}\left\{C_{0i} + \frac{A_i}{t_i^{k_i}}\right\} \tag{10}$$

- Rejection cost

In this study, produced components have a normal distribution with mean μ and the standard deviation of σ. As a result, under normality assumption in quality control, i.e. $\mathcal{N}(\mu_{x_i}, \sigma_{x_i})$, the probability of product rejection or the fraction of rejections can be estimated follows:

$$FR_{x_i} = 1 - \frac{1}{\sigma_{x_i}\sqrt{2\pi}} \int_{x_i{}^l}^{x_i{}^u} e^{-\frac{(x_i - \mu_{x_i})^2}{2\sigma_{x_i}^2}} dx_i. \tag{11}$$

The cost of product rejection can be estimated as the fraction of the manufacturing cost of the component which is called reject fraction (RF). Under normality assumption, components tolerances can be expressed based on *the K*-sigma concept ($t_{x_i} = K\sigma_{x_i} : K = 1,\ 2,\ \ 3$). The cost of product rejection of component x_i based on Eqs. 10 and 11 can be calculated as follows:

$$RC_{x_i} = FR_{x_i} \times MC_{x_i} = \left(1 - \frac{1}{\sigma_{x_i}\sqrt{2\pi}} \int_{-K\sigma_{x_i}}^{+K\sigma_{x_i}} e^{-\frac{1}{2}\left(\frac{t_{x_i}}{\sigma_{x_i}}\right)^2} dt_{x_i}\right) \left\{C_{0i} + \frac{A_i}{(t_{x_i})^{k_i}}\right\}. \tag{12}$$

The cost of product rejections for all components of assembly can be computed as below:

$$RC_{transmill} = \sum_{i=1}^{n} RC_{x_i}. \tag{13}$$

- *Inspection cost*

The inspection costs are dependent on several factors such as the inspection's methods, partial inspection, or total inspection. In this study, the inspection cost of each component is assumed constant cost (IC_{x_i}). Therefore, the total inspection cost of all components can be expressed as follows:

$$IC = \sum_{i=1}^{n} IC_{x_i}, \tag{14}$$

where n indicates the number of components of the assembly.

- *Total cost function*

According to all costs calculated, total cost includes the cost of quality reduction, manufacturing costs, rejection cost, and total inspection costs can be expressed as:

$$TC = RC + MC + QLC + IC. \tag{15}$$

- *Modelling functional characteristic in mechanical assembly*

The variation of functional characteristic can be defined as the greatest deviation from inference received by the designer in tolerance allocation as follows:

$$Y_v = f\left(\widetilde{t}_{x_1}, \ldots, \widetilde{t}_{x_n}\right). \tag{16}$$

- *Quality requirements of components*

To control the quality requirements of components, the process capability concept can be applied [39]. The index C_P is defined as

$$C_P = \frac{USL - LSL}{6\sigma}, \tag{17}$$

where *LSL* and *USL* are lower and upper specification limits of effective dimension x, respectively. Based on the 3-Sigma concept ($t = 3\sigma$), proper constraint to control quality requirement of components in optimum tolerance design can be rewritten as follows [39]:

$$C_{Pi} = \frac{USL_i - LSL_i}{6\sigma} \geq C_{min} \qquad i = 1, 2, \ldots, n, \tag{18}$$

where t_i indicates the tolerance of competent dimension i.

2.2.2.2. Formulating problem in multi-objective optimization form. According to modeled objective and cost functions, optimum tolerance allocation problem can be formulated as a multi-objective optimization problem as follows:

$$\begin{aligned} \text{Min } TC &= RC + MC + IC + QLC \\ &= K\left\{\left\{\sum_{i=1}^{n} (\theta_i t_{x_i})^2\right\}^{1/2} - t_{y_d}\right\}^2 + \sum_{i=1}^{n}\left\{C_{0i} + \frac{A_i}{(t_{x_i})^{k_i}}\right\} \\ &+ \sum_{i=1}^{n}\left\{\left(1 - \frac{1}{\sigma_{x_i}\sqrt{2\pi}} \int_{-K\sigma_{x_i}}^{+K\sigma_{x_i}} e^{-\frac{1}{2}\left(\frac{t_{x_i}}{\sigma_{x_i}}\right)^2} dt_{x_i}\right)\left\{C_{0i} + \frac{A_i}{(t_{x_i})^{k_i}}\right\}\right\} + \sum_{i=1}^{n} IC_{x_i}\ , \end{aligned}$$

$$\text{Min } Y_v = f\left(\widetilde{t}_{x_1}, \ldots, \widetilde{t}_{x_n}\right), \tag{19}$$

subject to

$$C_{Pi} = \frac{UL_i - LL_i}{2t_i} \geq 1 \qquad i = 1, 2, \ldots, n. \tag{20}$$

2.3. Extracting the non-dominated Pareto front of optimum tolerances for components

To determine optimum tolerances in the Pareto front form, the multi-objective optimization problem should be solved by a proper approach. In this study, Non-dominated Sorting Genetic Algorithm II (NSGA II), which was developed [40], as a powerful tool for solving multi-objective optimization problems (Eq. 19) is applied.

2.4. Selecting the best optimum tolerances from the obtained Pareto front

Optimum Pareto front obtained from solving multi-objective optimization problem (Eq. (19)) contains several sets of optimum tolerances that designers can select in the tolerance design stage. Decision-making for choosing the most preferred of optimum tolerances is not an easy and straightforward task. This challenge can be resolved by using a TOPSIS–based method developed [41].

In the proposed method, for choosing the most preferred of optimum tolerances Shannon's Entropy-based TOPSIS technique is applied [42]. According to this technique, the most preferred optimum solution can be chosen from the Pareto front by sorting optimum solutions with respect to closeness coefficients (C_i^*).

2.5. Bayesian reforming optimum tolerances to improve the reliability of the assembly

In the last step of the proposed method, the assembly reliability under optimum tolerances should be improved. Accordingly, based on design requirements, the limit state function(s) should be defined first. In this study, the limit state function (LSF) for the product at tolerance ($\tilde{t}_x$) is defined as follows:

$$g\left(\tilde{t}_x\right) = t_c - \tilde{t}_y, \tag{21}$$

where $\tilde{t}_y$ represents the Bayesian regression model of design function (Eq. (2)) and t_c is the critical limit of a performance characteristic of the product. Every reliability analysis method can be used to evaluate the assembly reliability in the proposed approach. In this study, the FROM using the gradient-based improved Hasofer-Lind and Rackwitz-Fiesler (iHLRF) searching algorithm [43] is utilized. If the obtained optimal tolerances do not satisfy the predefined reliability, importance vector should be used to increase the design reliability by shifting the tolerances toward the feasible/safe region.

An importance vector is a computational tool that determines the relative importance of the various parameters involved in the reliability analysis [44]. Importance vector $\tilde{\alpha}$ is an acceptable vector for random variables in normal standard space. In the case of variables are correlated variables, vector $\tilde{\alpha}$ has some error and for analyzing correlated random variables, the importance vector should be applied [44]. The normalized form as follows:

$$\tilde{\gamma} = \frac{\tilde{\alpha}^T J_{\tilde{t}_{y^*},\tilde{t}_{x^*}} D}{\| \tilde{\alpha}^T J_{\tilde{t}_{y^*},\tilde{t}_{x^*}} D \|}, \tag{22}$$

where D is a diagonal matrix of standard deviations and $J_{\tilde{t}_{y^*},\tilde{t}_{x^*}} = \partial\tilde{t}_{y^*} / \partial\tilde{t}_{x^*}$.

Accordingly, the normalized importance vector can be used to improve assembly reliability through the reforming procedure:

$$t^*_{xR} = t^*_x - \left(\gamma_{(x)} \times t^*_x\right), \tag{23}$$

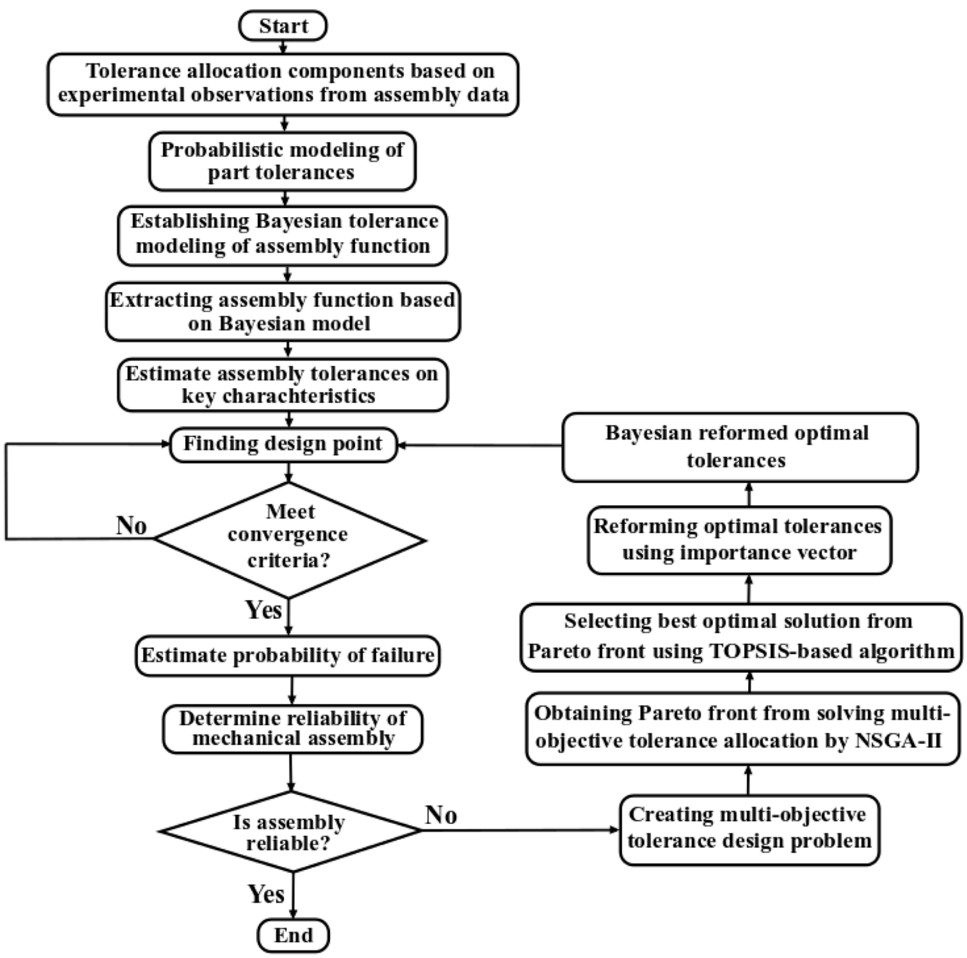

Fig. 1. The flowchart of the proposed method.

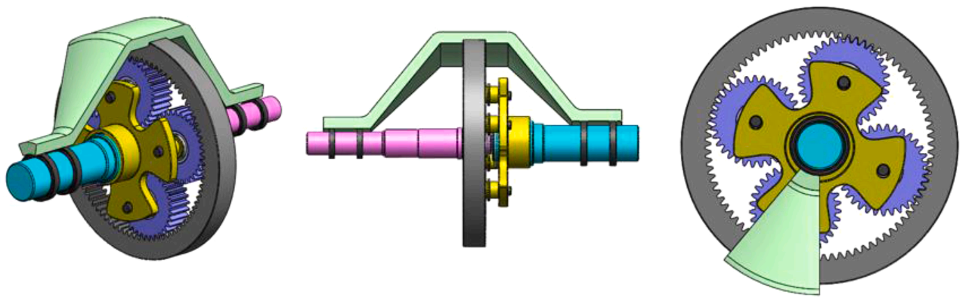

Fig. 2. Single-stage windmill planetary gear transmission system.

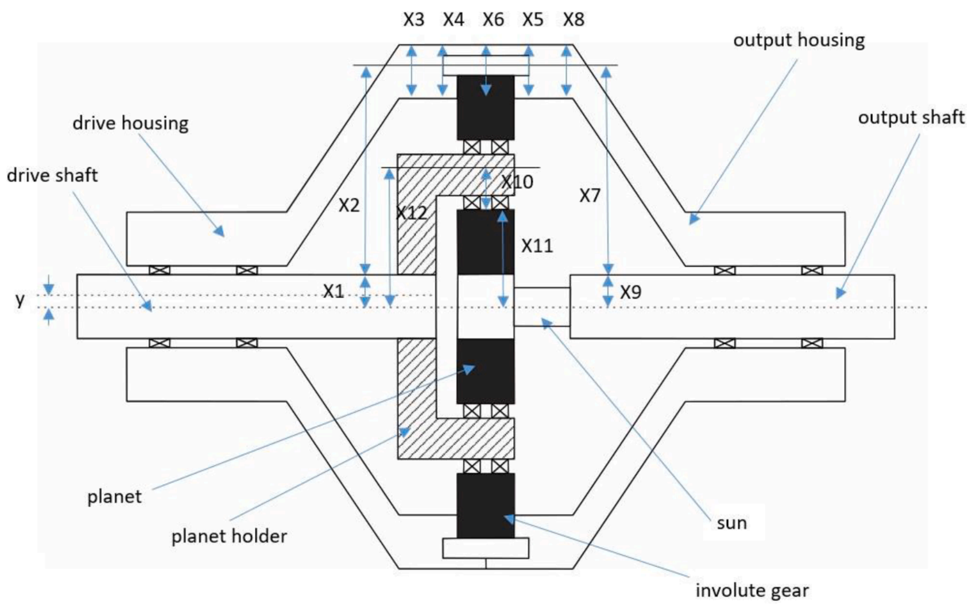

Fig. 3. Schematic of transmission planetary gear system.

where t_x^* and t_x^* indicate optimum tolerance vector before and after the reforming procedure. $\gamma_{(x)}$ indicates normalized importance vector.

For an overview of the main steps of the proposed method, its flowchart is illustrated in Fig. 1.

3. Illustrative case study

To demonstrate the applicability of the proposed method, a transmission planetary gear system as a case study from the literature [45] is considered (see Fig. 2).

The schematic of the transmission planetary gear system including its kinematic characteristics (x_1 to x_{12}) is shown in Fig. 3.

In this single-stage transmission system consisting of a drive and output housing, a drive and output shaft, a universal ring gear, three planets, and sun, sun movement relative to the planets results in excessive noise. Therefore, misalignment (t_y) between drive and output shafts is considered a functional characteristic.

In the following sub-sections, the main steps of the proposed method are implemented on the planetary gear system of a wind turbine.

3.1. Constructing Bayesian regression model of assembly function based on experimental results

In this study, the dimensional tolerances of the planetary gear system are modeled in the Normal distribution based on experimental observations. To achieve this aim, the experimental dataset is extracted from the available data in Ref.[45] Then, based on the extracted dataset, the variable inferences are applied to determine the synthetic probability density function (PDF) for the independent variables t_{x_i}. During this process, the synthetic PDF candidates are compared with known standard PDF, and the most appropriate PDF is selected as the synthetic PDF. In this way, the synthetic PDF of all tolerances are determined.

In general, there are two approaches for verifying the normality assumption; (1) Graphical approaches and numerical methods [46]. Graphical approaches as visual tools can present a visual comparison between the assumed distribution and the theatrical distribution in a plot (e.g. quantile (Q-Q) plots). For evaluating normality assumption, numerical methods can evaluate the normality assumption through descriptive statistics or statistical tests of normality (e.g. e Kolmogorov-Smirnov test) [47]. In this study for verifying the agreement of the distribution with the normality assumption, the quantile (Q-Q) plot as a graphical approach is used.

According to Fig. 4, synthetic PDF of effective part tolerances are in good agreement with the normal distribution.

Based on the proposed method, a Bayesian linear regression model can be created using experimental data. The corresponding Bayesian linear regression the model can be written as follows:

$$t_y = \theta_1 t_{x_1} + \theta_2 t_{x_2} + \theta_3 t_{x_3} + \theta_4 t_{x_4} + \theta_5 t_{x_5} + \theta_6 t_{x_6} + \theta_7 t_{x_7} + \theta_8 t_{x_8} + \theta_9 t_{x_9} + \theta_{10} t_{x_{10}} + \theta_{11} t_{x_{11}} + \theta_{12} t_{x_{12}} + \theta_{13} t_{x_{13}} + \varepsilon, \tag{24}$$

where t_y is misalignment and t_{x_i} are tolerance of design variables. θ_i indicates model parameters and ε is a model error with the normal distribution. The mean and Coefficient of variation of $\widetilde{\theta}$ are computed as

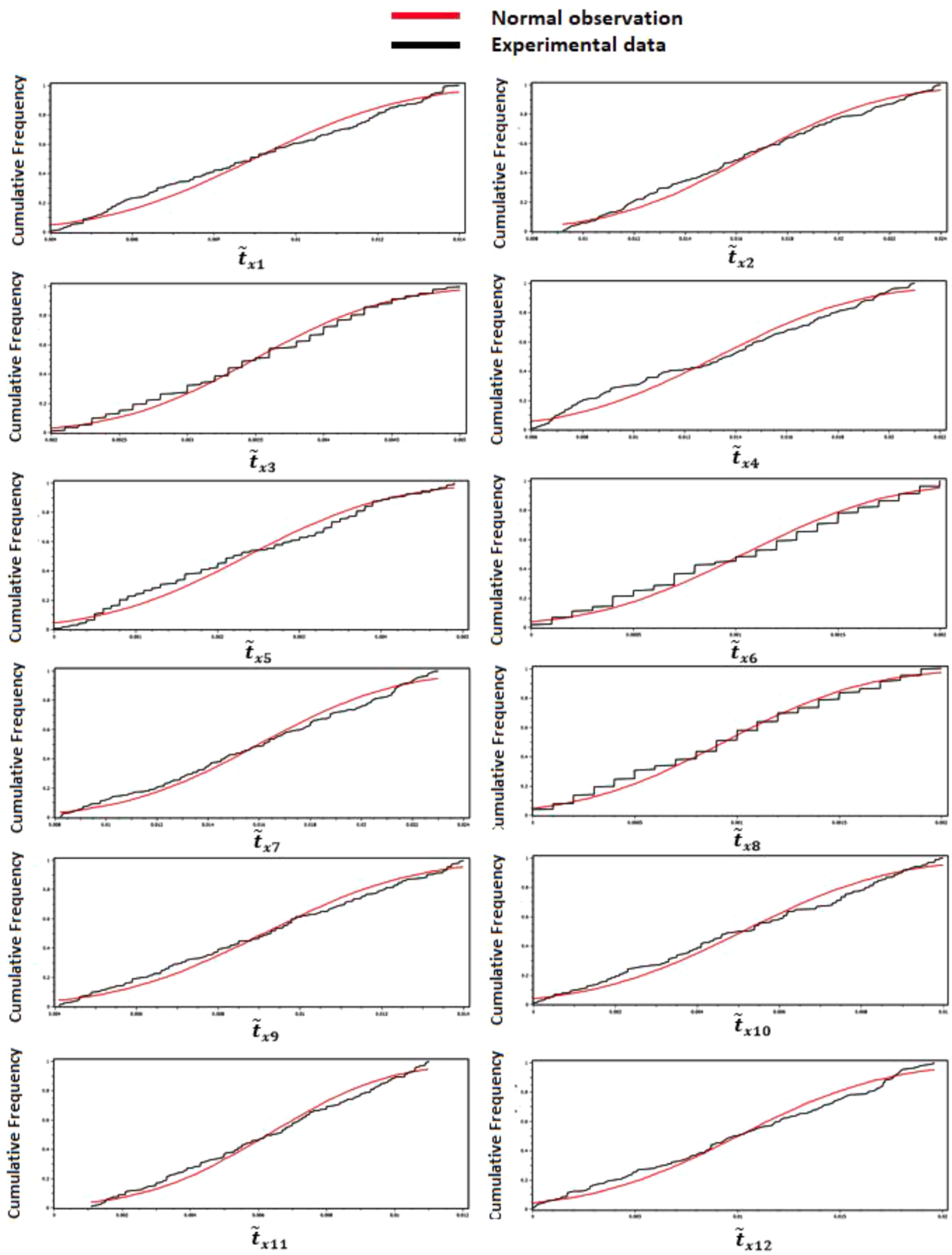

Fig. 4. Comparison of synthetic PDF of effective part tolerances and normal PDF.

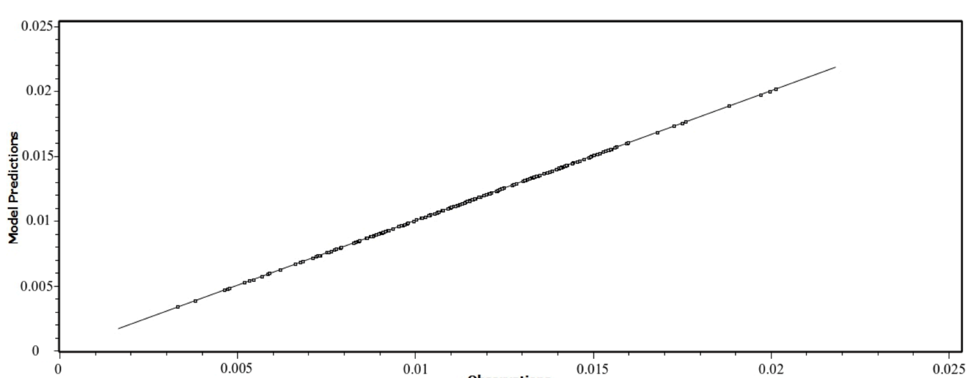

Fig. 5. Evaluating the normality of obtained Bayesian model.

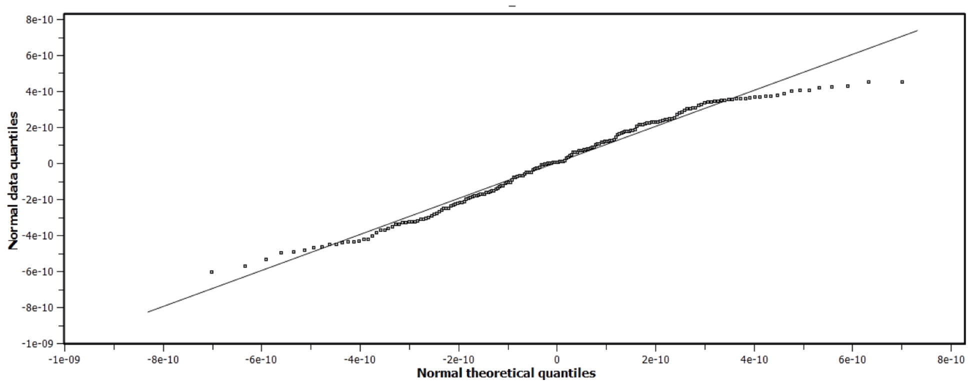

Fig. 6. Evaluating the normality of the probability distribution function of error (ε).

follows:

$$E(\widetilde{\theta}) = [0.321, 0.321, -0.321, -0.321, -0.321, 0.321, 0.321, 0.321, 0.321, 0.096, 0.096, -0.321, 0.003]. \tag{25}$$

Accordingly, correlation matrix can be obtained as follows:

$$cov(\theta_i, \theta_j) = \begin{bmatrix} 1 & 0.016380 & 0.16616 & -0.15997 & 0.01531 & -0.05175 & -0.02529 & -0.09074 & 0.06446 & 0.00088 & -0.02072 & -0.03141 & -0.33313 \\ 0.016380 & 1 & -0.01592 & -0.08481 & 0.02629 & 0.01622 & 0.05610 & 0.03253 & 0.04447 & -0.1516 & -0.07248 & -0.07993 & -0.36494 \\ 0.16616 & -0.01592 & 1 & -0.10856 & 0.08433 & -0.0295 & -0.05445 & 0.04091 & -0.08369 & 0.06028 & 0.04739 & 0.07355 & -0.48427 \\ -0.15997 & -0.08481 & -0.10856 & 1 & 0.02950 & 0.09374 & 0.06673 & -0.07823 & -0.07903 & -0.00124 & -0.11480 & -0.00335 & -0.15613 \\ 0.01531 & 0.02629 & 0.08433 & 0.02950 & 1 & 0.00359 & -0.09785 & -0.17171 & 0.06081 & -0.00721 & 0.00481 & 0.00804 & -0.19590 \\ -0.05175 & 0.01622 & -0.0295 & 0.09374 & 0.00359 & 1 & 0.00587 & 0.00974 & 0.05155 & 0.08975 & -0.03292 & 0.15702 & -0.25234 \\ -0.02529 & 0.05610 & -0.05445 & 0.06673 & -0.09785 & 0.00587 & 1 & -0.00842 & -0.04732 & 0.06102 & -0.02861 & 0.02712 & -0.37342 \\ -0.09074 & 0.03253 & 0.04091 & -0.07823 & -0.17171 & 0.00974 & -0.00842 & 1 & -0.10776 & -0.02495 & 0.11675 & 0.03382 & -0.11827 \\ 0.06446 & 0.04447 & -0.08369 & -0.07903 & 0.06081 & 0.05155 & -0.04732 & -0.10776 & 1 & 0.02415 & -0.07944 & -0.00850 & -0.2721 \\ 0.00088 & -0.15161 & 0.06028 & -0.00124 & -0.00721 & 0.08975 & 0.06102 & -0.02495 & 0.02415 & 1 & 0.04439 & 0.08161 & -0.21868 \\ -0.02072 & -0.07248 & 0.04739 & -0.1148 & 0.00481 & -0.03292 & -0.02861 & 0.11675 & -0.07944 & 0.04439 & 1 & 0.04228 & -0.17138 \\ -0.03141 & -0.07993 & 0.07355 & -0.00335 & 0.00804 & 0.15702 & 0.02712 & 0.03382 & -0.00850 & 0.08161 & 0.04228 & 1 & -0.24247 \\ -0.33313 & -0.36494 & -0.48427 & -0.15613 & -0.19590 & -0.25234 & -0.37342 & -0.11827 & -0.2721 & -0.21868 & -0.17138 & -0.24247 & 1 \end{bmatrix} \tag{26}$$

Consequently, error distribution is obtained as follows:

$$\varepsilon : \mathcal{N}(0,\ 0.05227). \tag{27}$$

To evaluate the obtained Bayesian linear regression model, corresponding R-factor and normality can be computed. The computed R-factor of the Bayesian linear regression model is 0.992, which illustrates a good agreement between the obtained model and experimental data. Figs. 5 and 6, show extracted Bayesian model and distribution function of error (ε) are in good agreement with the normality assumption, respectively.

Consequently, the relationship between misalignment as functional characteristic (t_y) and effective tolerances (t_{x_2}) can be expressed as below

$$t_y = 0.321t_{x_1} + 0.321t_{x_2} - 0.321t_{x_3} - 0.321t_{x_4} - 0.321t_{x_5} + 0.321t_{x_6} + 0.321t_{x_7} + 0.321t_{x_8} + 0.321t_{x_9} + 0.096t_{x_{10}} + 0.096t_{x_{11}} - 0.321t_{x_{12}} + 0.003, \tag{28}$$

Under the normality assumption, components tolerances can be expressed based on the 3-sigma concept:

$$t_{x_i} = 3\sigma_{x_i}, \qquad i = 1, 2, \ldots, 12, \tag{29}$$

- Estimating product reliability to meet functional requirements based on FORM method

In this step, the assembly reliability to meet the quality requirement is obtained using FORM method. To reach this aim, limit state function can be considered as follows:

$$g\left(\widetilde{t}_x\right) = 0.01 - y\left(\widetilde{t}_x\right), \tag{30}$$

where $\widetilde{t}_y$ is obtained Bayesian linear regression model from Eq. 28 and quality requirement to control key characteristic i.e. the misalignment of planetary gear is 0.01. In the following, using the FORM method, the reliability level is 32.04%.

- Multi-objective optimum tolerance design

Referring to Eqs. (19) and (20), the multi-objective optimum

Table 1
Statistical specifications of effective dimensions and parameters.

Effective Dimension	μ_x (mm)	σ_x(mm)	LL_{x_i}(mm)	UL_{x_i} (mm)	C_0	A	k	m_x($)	IC_x($)	C_{min}
x_1	40	0.006	39.982	40.018	1.5	0.063	1	5	0.25	1.5
x_2	173.33	0.009	173.303	173.357	2	0.081	1	5	0.25	1.5
x_3	53.33	0.002	53.324	53.336	1.5	0.021	1	5	0.25	1.5
x_4	53.33	0.002	53.324	53.336	1.7	0.0198	1	5	0.25	1.5
x_5	53.33	0.002	53.315	53.345	2	0.018	1	5	0.25	1.5
x_6	20	0.001	19.997	20.003	1	0.012	1	5	0.25	1.5
x_7	173.33	0.009	173.303	173.357	2	0.081	1	5	0.25	1.5
x_8	53.33	0.002	53.324	53.336	1.3	0.022	1	5	0.25	1.5
x_9	40	0.006	39.982	40.018	1.5	0.063	1	5	0.25	1.5
x_{10}	80	0.005	79.985	80.015	2	0.045	1	5	0.25	1.5
x_{11}	40	0.005	39.985	40.015	2	0.045	1	5	0.25	1.5
x_{12}	60	0.009	59.973	60.027	1.7	0.089	1	5	0.25	1.5

tolerance design can be formulated for this study as follows:

$$\min\ TC = K\Big\{\big\{(0.321t_{x_1})^2 + (0.321t_{x_2})^2 + (-0.321t_{x_3})^2 + (-0.321t_{x_4})^2 + (-0.321t_{x_5})^2 + (0.321t_{x_6})^2$$
$$+(0.321t_{x_7})^2 + (0.321t_{x_8})^2 + (0.321t_{x_9})^2 + (0.096t_{x_{10}})^2 + (0.096t_{x_{11}})^2 + (-0.321t_{x_{12}})^2 + (0.003)^2\big\}^{1/2} - t_{yd}\Big\}^2 +$$
$$\sum_{i=1}^{12}\left\{C_{0i} + \frac{A_i}{(t_{x_i})^{k_i}}\right\} + \sum_{i=1}^{12}\left\{\left(1 - \frac{1}{\sigma_{x_i}\sqrt{2\pi}}\int_{-3\sigma_{x_i}}^{+3\sigma_{x_i}} e^{-\frac{1}{2}\left(\frac{t_{x_i}}{\sigma_{x_i}}\right)^2} dt_{x_i}\right)\left\{C_{0i} + \frac{A_i}{(t_{x_i})^{k_i}}\right\}\right\} + \sum_{i=1}^{12} IC_{x_i}$$

$$\min\ t_y = 0.003 + 0.321t_{x_1} + 0.321t_{x_2} + 0.321t_{x_3} + 0.321t_{x_4} + 0.321t_{x_5} + 0.321t_{x_6} + 0.321t_{x_7} + 0.321t_{x_8} + 0.321t_{x_9} + 0.096t_{x_{10}} + 0.096t_{x_{11}} + 0.321t_{x_{12}} \quad (31)$$

Subject to:

$$C_{Pi} = \frac{USL_i - LSL_i}{2\ t_{x_i}} \geq C_{min}, \qquad i = 1, 2, \ldots, 12, \quad (32)$$

where all values of parameters in Eq. (31) are reported in Table 1. Also, in this study, the desired misalignment (t_y) should be 0.01.

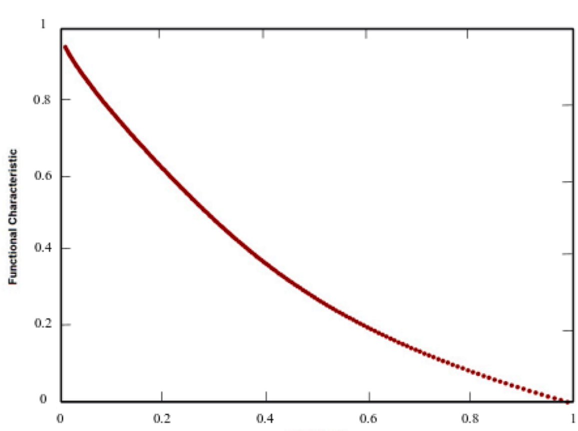

Fig. 7. The obtained Pareto front from NSGAII as trade-off frontier between normalized conflicting objectives space: total cost and functional characteristic.

- Obtaining the non-dominated Pareto front of optimum tolerances for components

For solving the bi-objective optimum tolerance design problem (Eq. (31)), the NSGA II method is utilized. The parameters of NSGA-II are adjusted in this study as follows: population size: 190 individuals, generation number: 2000 generations, crossover rate: 0.8, mutation rate: 0.005. Solving bi-objective tolerance design problem (Eq. (31)) through NSGA-II results from Pareto front, which is shown in Fig. 7.

- Selecting the best optimum tolerances from obtained Pareto front

Using Shannon's Entropy-based TOPSIS method, 190 optimum solutions on the obtained Pareto front as candidates are sorted with respect to closeness coefficients (see Fig. 8). In this study, candidate 135 (S_{135}) with the highest closeness coefficient ($C^* = 0.667$) is selected as the most preferred optimum tolerances. Consequently, the most preferred optimum tolerances are reported in Table 2.

Base on the best optimum tolerances set (S_{40}), the optimum total cost is 57.056$ and the misalignment of the main shafts as a functional characteristic is 0.0101 *mm* and assembly reliability is 94.87%.

- *Bayesian reforming optimum tolerances to improve the reliability of the assembly*

Since the obtained reliability of assembly under the conventional tolerances is at a non-acceptable level (i.e. 32.03%), according to Eq. (22), the normalized importance vector of γ, which determines the importance of each tolerance in assembly reliability, can be computed as follows:

Furthermore, to demonstrate the effectiveness of the proposed method, the obtained results from the proposed method and results of a traditional tolerance synthesis approach proposed in Ref. [49] are compared. Components tolerances of the gearbox assembly under different conditions (i.e. conventional tolerances, the optimum

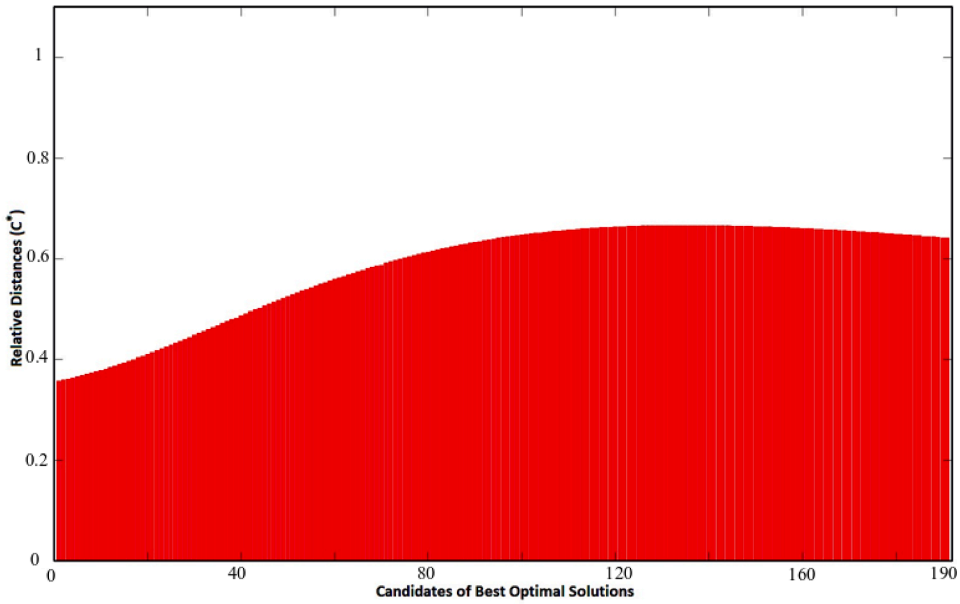

Fig. 8. The relative distances of 190 candidates of best optimum based on TOPSIS technique.

tolerances obtained from a traditional tolerance synthesis method proposed in [48], the optimum tolerances from the proposed method without and with the Bayesian reforming procedure) are reported in Table 2.

Regarding the proposed algorithm, modification of mean values of component tolerances t_{x_i} is performed in the regression model. After this correction, a new model of reliability analysis is carried out based on the proposed method.

3.2. Verification and discussion

To verify the proposed method, the Monte Carlo simulation method is the only method in the literature that both tolerance and reliability analysis can be applied [49]. Therefore, it can be an appropriate technique for comparing with the proposed method. For verifying the obtained results of assembly reliability analysis from the proposed method, results of the proposed method are compared to results of Monte Carlo simulations. Therefore, the assembly reliability value of gearbox assembly is estimated using 300000 simulations of the Monte Carlo method at the same condition under initial and optimum tolerances. CDF, PDF, and the covariance (*CoV*) of the Monte Carlo simulations under initial and optimum tolerances are shown in Figs. 9 and 10, respectively.

Compared to the obtained results of the proposed method and Monte Carlo simulation are reported in Table 3. According to the obtained results, the reliability of assembly based on the proposed method and Monte Carlo simulations under conventional tolerances for limit state 0.01 is 32.03% and 33.43%, respectively.

In other words, in this condition, the relative error of the obtained result from the proposed method in comparison to Monte Carlo simulations is 4.37%. Referring to Table 3, the relative errors of estimated reliability values under optimum tolerances before and after reforming procedure from the proposed method in comparing to Monte Carlo simulations are 0.26 % and 0.06%, respectively. Consequently, the results of the proposed method under low computational time are in good agreement with the accurate results of the Monte Carlo simulation approach as a time-consuming and computationally intensive method.

To illustrate the capability of the proposed method, design criteria under conventional tolerances and optimum tolerances without and with Bayesian reforming procedure are compared which are reported in Table 4.

Referring to Table 4, the total cost under optimum tolerances from the proposed method before and after Bayesian reforming with respect to the conventional condition is reduced by 45.1% and 36.4%, respectively. Also, mean and standard deviation values (μ_y, σ_y) of functional characteristics under optimum tolerances without and with Bayesian reforming with respect to the conventional condition are tightened by (98.1%, 82.4%) and (99.9%, 100%), respectively. Accordingly, assembly reliability (R) under optimum tolerances from the proposed method before and after Bayesian reforming with respect to the conventional condition is improved by 196.2% and 211.9%, respectively. Referring to computational results, although using the proposed Bayesian procedure increases total cost by 15.8%, both functional characteristic(μ_y, σ_y), and the assembly reliability (R) are improved by (95.4%, 100%) and 5.3%, respectively.

Moreover, the obtained results from the traditional tolerance synthesis method proposed in [48] are also presented in Table 4. Referring to Table 4, compared with the tolerance synthesis approach in Ref. [48], the total cost is decreased by 24.67% and 12.79% in the proposed approach before and after Bayesian reforming, respectively. This is

Table 2
Component tolerances of the gearbox assembly.

Tolerances	Conventional tolerances	Optimum tolerances from Muthu method [48]	Optimum tolerances from the proposed method	
			without Bayesian reforming	with Bayesian reforming
t_{x1}	0.0089	0.0068	0.0020	0.0014
t_{x2}	0.0163	0.0140	0.0090	0.0054
t_{x3}	0.0034	0.0021	0.0030	0.0032
t_{x4}	0.0034	0.0022	0.0040	0.0057
t_{x5}	0.0023	0.0018	0.0087	0.0098
t_{x6}	0.0010	0.0014	0.0020	0.0019
t_{x7}	0.0160	0.013	0.0060	0.0036
t_{x8}	0.0009	0.0009	0.0010	0.0009
t_{x9}	0.0091	0.0038	0.0020	0.0014
t_{x10}	0.0051	0.0040	0.0020	0.0018
t_{x11}	0.0062	0.0072	0.0090	0.0083
t_{x12}	0.0100	0.0080	0.0050	0.0077

All tolerances are in millimeters (mm)

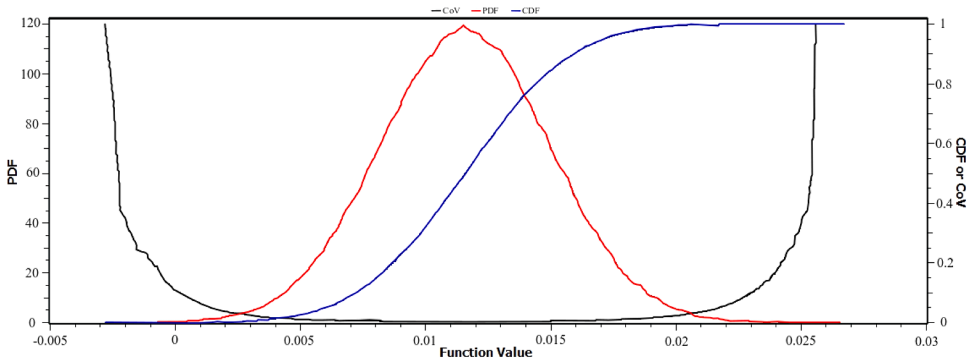

Fig. 9. Cumulative distribution function (CDF), probability density function (PDF), and covariance (Cov) of Monte Carlo simulations under conventional tolerances.

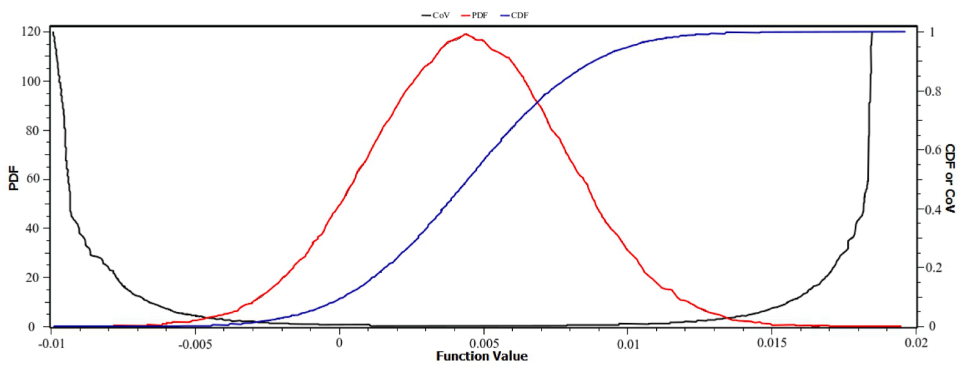

Fig. 10. Cumulative distribution function (CDF), probability density function (PDF), and covariance (CoV) of Monte Carlo simulations under the optimum tolerances from the proposed method.

Table 3
Comparing the assembly reliability from the proposed method and Monte Carlo simulations under different conditions.

Conditions		Assembly reliability (***R***%)		Relative error
		Proposed method	Monte Carlo simulations	
Under conventional tolerances		32.03%	33.43%	4.37%
Under optimum tolerances from the proposed method	without Bayesian reforming	94.87%	95.12%	0.26 %
	with Bayesian reforming	99.91%	99.97%	0.06%

because the conventional method takes into account only the manufacturing and quality loss costs and discards rejection and inspection costs. In contrast to the conventional approach, in which a deterministic constraint is defined on the quality characteristic, the quality characteristic is considered as an additional cost function in the proposed approach.

Therefore, the proposed approach leads to a more reliable design with respect to the conventional tolerance synthesis approach. Also, it is worth noting that the proposed approach takes into account the epistemic uncertainty due to approximations in the system behavior model using the Bayesian regression model, and hence, the results obtained from it are more accurate and reliable than the obtained results from the conventional approaches.

Consequently, based on obtained results, using the proposed method can simultaneously lead to decrees total cost, increase the quality of functional characteristic, and improve assembly reliability.

Table 4
Comparing obtained results from the proposed method and conventional approach.

Design Criteria	Under conventional tolerances	Under Muthu method [48]	Under optimum tolerances from the proposed method	
			Without Bayesian reforming	With Bayesian reforming
Total Cost ($)	57.056	41.58	31.319	36.263
Functional characteristic $\mathbf{t_y}:\mathcal{N}(\mu_y,\ \sigma_y)$ (mm)	$\mathcal{N}$(0.116, 0.0034)	$\mathcal{N}$(0.0099, 0.0008)	$\mathcal{N}$(0.0022, 0.0006)	$\mathcal{N}$(0.0001, 0.0000)
Assembly reliability (***R***)	32.03%	50.16%	94.87%	99.91%

4. Conclusions

Despite the importance of product reliability evaluation, researchers in literature have not previously considered the tolerance - reliability design of mechanical assemblies. In order to obtain optimal tolerances satisfying desired reliability, this paper proposed a reliability-based optimal tolerance design method. The proposed method approximates the assembly function based on experimental observations by Bayesian linear regression. As a result, this method can be applied efficiently to the tolerance design of complex assemblies where the explicit assembly functions are difficult, if not impossible, to extract. Also, using Bayesian linear regression allows the designer to deal with meta-modeling uncertainties. Then, with the aim of assuring optimality and quality, the tolerance allocation problem is formulated as a bi-objective optimization problem that contains minimizing total cost (summation of quality loss and production costs) and minimizing the variation of functional characteristics. Then, the proposed method employs the NSGA-II multi-objective optimization approach to find Non-dominated optimal solutions. In order to select the best tolerances from the Pareto front, this approach utilizes Shannon's entropy-based TOPSIS algorithm, which is a powerful multi-criteria decision-making tool. Finally, in order to increase the reliability to the desired level, the importance vector is used to correct obtained optimal tolerances.

To demonstrate the capability of the proposed method, a transmission planetary gear system was considered as an illustrative case study. According to obtained optimum tolerances, using the proposed method can concurrently reduce total cost, increase the quality of functional characteristics, and improve the assembly reliability of the product.

CRediT authorship contribution statement

A. Ghaderi: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curtion, Writing – original draft, Visualization. **H. Hassani:** Conceptualization, Methodology, Investigation, Writing – review & editing, Visualization. **S. Khodaygan:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing – review & editing, Visualization, Supervision, Project administration.

Declaration of Competing Interest

None.

References

[1] Cao Y, Liu T, Yang J. A comprehensive review of tolerance analysis models. Int J Adv Manuf Technol 2018;97(5-8):3055–85.
[2] Chen H, et al. A comprehensive study of three dimensional tolerance analysis methods. Comput Aided Des 2014;53:1–13.
[3] Singh P, Jain P, Jain S. Important issues in tolerance design of mechanical assemblies. Part 1: tolerance analysis. Proc Inst Mech Eng Part B J Eng Manuf 2009;223(10):1225–47.
[4] Chase KW, Gao J, Magleby SP. General 2-D tolerance analysis of mechanical assemblies with small kinematic adjustments. J Des Manuf 1995;5:263–74.
[5] Laperrière L, Lafond P. Tolerance analysis and synthesis using virtual joints. Global Consistency of Tolerances. Springer; 1999. p. 405–14.
[6] Clemént A, et al. The TTRSs: 13 constraints for dimensioning and tolerancing. Geometric design tolerancing: theories, standards and applications. Springer; 1998. p. 122–31.
[7] Desrochers A, Ghie W, Laperriere L. Application of a unified Jacobian—Torsor model for tolerance analysis. J Comput Inf Sci Eng 2003;3(1):2–14.
[8] Khodaygan S, Movahhedy M, Fomani MS. Tolerance analysis of mechanical assemblies based on modal interval and small degrees of freedom (MI-SDOF) concepts. Int J Adv Manuf Technol 2010;50(9-12):1041–61.
[9] Ziegler P, Wartzack S. Sensitivity analysis of features in tolerancing based on constraint function level sets. Reliab Eng Syst Saf 2015;134:324–33.
[10] Khodaygan S, Movahhedy M. Tolerance analysis of assemblies with asymmetric tolerances by unified uncertainty–accumulation model based on fuzzy logic. Int J Adv Manuf Technol 2011;53(5-8):777–88.
[11] Khodaygan S, Movahhedy M. A comprehensive fuzzy feature-based method for worst case and statistical tolerance analysis. Int J Comput Integr Manuf 2016;29(1):42–63.
[12] Khodaygan S, Ghaderi A. Tolerance–reliability analysis of mechanical assemblies for quality control based on Bayesian modeling. Assembly Automation; 2019.
[13] Homri L, et al. Tolerance analysis—Form defects modeling and simulation by modal decomposition and optimization. Comput Aided Des 2017;91:46–59.
[14] Liu T, et al. Assembly tolerance analysis based on the Jacobian model and skin model shapes. Assem Autom 2019.
[15] Tlija M, et al. A Novel Model for the Tolerancing of Nonrigid Part Assemblies in Computer Aided Design. J Comput Inf Sci Eng 2019;19(4).
[16] Tlija M, Ghali M, Aifaoui N. Integrated CAD tolerancing model based on difficulty coefficient evaluation and Lagrange multiplier. Int J Adv Manuf Technol 2019;101(9-12):2519–32.
[17] Corrado A, et al. A variational model for 3D tolerance analysis with manufacturing signature and operating conditions. Assem Autom 2018.
[18] Polini W, Corrado A. A Geometric Model for Tolerance Analysis with Manufacturing Signature and Operating Conditions. Int J Manuf, Mater Mech Eng (IJMMME) 2019;9(3):1–13.
[19] Corrado A, Polini W. Comparison among different tools for tolerance analysis of rigid assemblies. Int J Comput Appl Technol 2020;62(1):36–44.
[20] Anwer N, Ballu A, Mathieu L. The skin model, a comprehensive geometric model for engineering design. CIRP Ann 2013;62(1):143–6.
[21] Kong X, Yang J, Hao S. Reliability modeling-based tolerance design and process parameter analysis considering performance degradation. Reliab Eng Syst Saf 2020:107343.
[22] Armillotta A. Selection of parameters in cost-tolerance functions: review and approach. Int J Adv Manuf Technol 2020;108:167–82.
[23] Huang YM, Shiau CS. An optimal tolerance allocation model for assemblies with consideration of manufacturing cost, quality loss and reliability index. Assem Autom 2009.
[24] Peng H. Concurrent tolerancing for design and manufacturing based on the present worth of quality loss. Int J Adv Manuf Technol 2012;59(9-12):929–37.
[25] Liu S, et al. A closed-form method for statistical tolerance allocation considering quality loss and different kinds of manufacturing cost functions. Int J Adv Manuf Technol 2017;93(5):2801–11.
[26] Natarajan J, Sivasankaran R, Kanagaraj G. Bi-objective optimization for tolerance allocation in an interchangeable assembly under diverse manufacturing environment. Int J Adv Manuf Technol 2018;95(5-8):1571–95.
[27] Khodaygan S. A multiple objective framework for optimal asymmetric tolerance synthesis of mechanical assemblies with degrading components. Int J Adv Manuf Technol 2019;100(9-12):2177–205.
[28] Khodaygan S. Meta-model based multi-objective optimisation method for computer-aided tolerance design of compliant assemblies. Int J Comput Integr Manuf 2019;32(1):27–42.
[29] Liu X, et al. A modified quality loss model of service life prediction for products via wear regularity. Reliab Eng Syst Saf 2020;204:107187.
[30] Hassani H, Khodaygan S, Ghaderi A. Bayesian reliability-based robust design optimization of mechanical systems under both aleatory and epistemic uncertainties. Eng Optimiz 2021.
[31] Vahidi-Moghaddam A, Rajaei A, Ayati M. Disturbance-observer-based fuzzy terminal sliding mode control for MIMO uncertain nonlinear systems. Appl Math Modell 2019;70:109–27.
[32] Vahidi-Moghaddam A, Mazouchi M, Modares H. Memory-augmented system identification with finite-time convergence. IEEE Control Syst Lett 2020;5(2):571–6.
[33] Turkman MAA, Paulino CD, Müller P. Computational Bayesian Statistics: An Introduction, 11. Cambridge University Press; 2019.
[34] Cheng K, Lu Z. Adaptive Bayesian support vector regression model for structural reliability analysis. Reliab Eng Syst Saf 2021;206:107286.
[35] Lee, W.-J. and T. Woo, *Optimum selection of discrete tolerances.*1989.
[36] Chase KW, Greenwood WH. Design issues in mechanical tolerance analysis. Manufac Rev 1988;1(1):50–9.
[37] Arora JS. Introduction to optimum design. Elsevier; 2004.
[38] Chase KW. Minimum cost tolerance allocation. Dimensioning and tolerancing handbook. 1999.
[39] Pearn WL, Kotz S. Encyclopedia and handbook of process capability indices: a comprehensive exposition of quality control measures. World Scientific; 2006.
[40] Deb K, et al. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Comput 2002;6(2):182–97.
[41] Yoon KP, Hwang C-L. Multiple attribute decision making: an introduction, 104. Sage publications; 1995.
[42] Deng H, Yeh C-H, Willis RJ. Inter-company comparison using modified TOPSIS with objective weights. Comput Oper Res 2000;27(10):963–73.
[43] Zhang Y, Der Kiureghian A. *Two improved algorithms for reliability analysis*, in *Reliability and optimization of structural systems*. Springer; 1995. p. 297–304.
[44] Kala Z. Global sensitivity analysis of reliability of structural bridge system. Eng Struct 2019;194:36–45.
[45] Gerth RJ, Pfeifer T. Minimum cost tolerancing under uncertain cost estimates. IIE Trans 2000;32(6):493–503.
[46] Baghban AA, et al. How to test normality distribution for a variable: a real example and a simulation study. Arch Adv Biosci 2013;4(1).
[47] Wilk MB, Gnanadesikan R. Probability plotting methods for the analysis for the analysis of data. Biometrika 1968;55(1):1–17.
[48] Muthu P, Dhanalakshmi V, Sankaranarayanasamy K. Optimal tolerance design of assembly for minimum quality loss and manufacturing cost using metaheuristic algorithms. Int J Adv Manuf Technol 2009;44(11-12):1154–64.
[49] Alban A, et al. Efficient Monte Carlo methods for estimating failure probabilities. Reliab Eng Syst Saf 2017;165:376–94.