Tolerance–reliability analysis of mechanical assemblies for quality control based on Bayesian modeling

S. Khodaygan and A. Ghaderi
Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

Abstract

Purpose – The purpose of this paper is to present a new efficient method for the tolerance–reliability analysis and quality control of complex nonlinear assemblies where explicit assembly functions are difficult or impossible to extract based on Bayesian modeling.

Design/methodology/approach – In the proposed method, first, tolerances are modelled as the random uncertain variables. Then, based on the assembly data, the explicit assembly function can be expressed by the Bayesian model in terms of manufacturing and assembly tolerances. According to the obtained assembly tolerance, reliability of the mechanical assembly to meet the assembly requirement can be estimated by a proper first-order reliability method.

Findings – The Bayesian modeling leads to an appropriate assembly function for the tolerance and reliability analysis of mechanical assemblies for assessment of the assembly quality, by evaluation of the assembly requirement(s) at the key characteristics in the assembly process. The efficiency of the proposed method by considering a case study has been illustrated and validated by comparison to Monte Carlo simulations.

Practical implications – The method is practically easy to be automated for use within CAD/CAM software for the assembly quality control in industrial applications.

Originality/value – Bayesian modeling for tolerance–reliability analysis of mechanical assemblies, which has not been previously considered in the literature, is a potentially interesting concept that can be extended to other corresponding fields of the tolerance design and the quality control.

Keywords Reliability analysis, Tolerance analysis, Bayesian modeling, Mechanical assembly

Paper type Research paper

1. Introduction

Manufacturing processes are not inherently precise processes. Several effective factors such as tool wear, tool vibration, and other factors cause deviations of the product from the nominal design condition. To define and communicate the dimensional and geometric tolerances, Geometric dimensioning and tolerancing (GD&T) as a symbolic language has been developed. GD&T is a useful tool to specify the requirements related to the design of components, which takes into account the functional and assembly criteria (Polini, 2016). Correctly applying it can provide an efficient and relatively low-cost production of the designed parts (Polini, 2016). Since parts are most often used as an assembly, due to the accumulation of dimensional and geometrical deviations, the functionality of the mechanical assembly may be disturbed. The study on the effects of the propagation and the accumulation of tolerances in part and the assembly levels is called the tolerance analysis. Using tight tolerances increases the production cost of production and allocating the loose tolerances reduces the reliability of the mechanical assemblies. In other words, the tolerance analysis can be a bridge between the design and the manufacturing stages (Chase and Parkinson, 1991).

The several studies have been conducted to develop the tolerance analysis of mechanical assemblies. Wade (1983) presented the tolerance charting method for the tolerance analysis of mechanical assemblies. In the tolerance charting method, the specification of the effective dimensions is modeled by the one-dimensional vectors in the form of tolerance charts (Wade, 1983). In the parametric tolerance analysis, the design dimensions are expressed as the analytic functions of independent variables based on the parametric constraints governing the key characteristics (Pasupathy *et al.*, 2003). Requicha proposed the offset model for tolerance analysis (Requicha, 1983). In the offset model, the tolerance region is defined as the Boolean subtraction of the maximum and minimum volumes of the part. Consequently, the offset space can be considered as the part tolerances. The degrees of freedom (DOF) method was proposed by Bernstein (Bernstein, 1989). For tolerance modeling, the DOF method is established upon six levels of the plate, cylindrical, spherical, spiral, rotary and prism. These six geometric entities, in addition to the constraints governing the rigid body, are called

The current issue and full text archive of this journal is available on Emerald Insight at: **www.emeraldinsight.com/0144-5154.htm**

Assembly Automation
39/5 (2019) 769–782
© Emerald Publishing Limited [ISSN 0144-5154]
[DOI 10.1108/AA-06-2018-081]

Received 16 October 2017
Revised 16 January 2018
15 March 2018
10 June 2018
25 June 2018
29 July 2018
Accepted 3 August 2018

Seven Levels (TTRS) (Jayaraman and Srinivasan, 1989). Researchers presented a method for the tolerance analysis using small kinematic adjustment of components based on the linear approximation of implicit dimensional constraints which is called the direct linearization method (DLM) and the vector-loops model (Chase *et al.*, 1995). In the vector loop model, the dimensional and geometrical tolerances are described by a combination of vector loops and kinematic joints. Researchers presented a kinematics-based method for tolerance analysis and synthesis which uses Jacobian transform (Laperrière and Lafond, 1999). This method, in the tolerance modeling, considering all small displacements to geometric features which are known to affect some functional requirements. Davidson and Shah presented the tolerance analysis of mechanical assemblies based on the Tolerance-Map (T-Map) method (Davidson *et al.*, 2002). A T-Map is a hypothetical Euclidean point-space, its size and shape reflect all dimensional and geometric variational possibilities of the corresponding feature. Each is the range of points that result from a one-to-one mapping from all the variational possibilities of a feature.

Khodaygan *et al.* introduced a feature-based approach to tolerance analysis for mechanical assemblies with geometrical and dimensional tolerances (Khodaygan *et al.*, 2010). In this approach, the geometrical and dimensional tolerances are expressed by small degrees of freedom (SDOF) of geometric entities that are described by tolerance zones. Armillotta presented an approach method to relate the sensitivities of linearized functional equations to the free-body diagrams of force analysis (Armillotta, 2014). Based on this methodology, the tolerance chain problem can be converted to a static problem, which can be solved by conventional methods of force analysis. Khodaygan and Movahhedy proposed a comprehensive method for tolerance analysis of mechanical assemblies based on the Fuzzy–SDOF model (Khodaygan and Movahhedy, 2016). In this work, tolerances are described by an integrated fuzzy – SDOF model. Polini *et al.* proposed a method based on the Jacobin model for the analytical tolerance analysis of mechanical assemblies with geometric tolerances (Polini and Corrado, 2016). To reduce the generation of truncation errors, the method is established upon the nonlinear stack-up functions which relate the functional assembly requirements to the manufacturing variations. In the literature, tolerance accumulation analysis approaches can be classified into worst-case and statistical or root sum square (RSS) methods. These two methods were developed by Greenwood and Chase (Greenwood and Chase, 1988, Greenwood and Chase, 1990) for the linear analysis of nonlinear assembly functions and corrected by Khodaygan and Movaheedy for analyzing the asymmetric tolerances (Khodaygan and Movahhedy, 2011).

The reliability analysis of a component or a system refers to the probability of satisfying a performance criterion at the component or the system level, respectively. For reliability analysis, a multi-dimensional integral over the failure domain of the performance function should be computed (Madsen *et al.*, 2006). The major computational methods to solve this problem can be categorized into approaches; simulation-based methods and analytical approximations (Rebba and Mahadevan, 2008).

In the literature, several simulation-based approaches have been proposed such as direct Monte Carlo simulations (MC) (Dimov, 2008) and Importance Sampling (IS) (Gelman and Meng, 1998, Liu, 1996). In general, the simulation-based methods due to the large numbers of simulations are the time-consuming approaches. To overcome this drawback, the analytical approximation methods, such as the first-order reliability method (FORM) has been proposed. The FORM was firstly developed (Hasofer and Lind, 1974). The FORM with lesser computational time can be considered as a proper alternative to the simulation-based methods. The FORM can be used based on linear approximations of the limit state function (LSF) at the most probable point (MPP) (Bucher and Bourgund, 1990). Due to the capability of FORM in handling the non-linear performance functions, and correlated non-normal variables with proper accuracy and low computational time, some researches have been conducted based on it in the literature. Du and Hu proposed a modified FORM to evaluate the reliability where the probability distributions of some random variables are truncated (Du and Hu, 2012). According to this method, the truncated random variables are transformed into truncated standard normal variables, and then the reliability is estimated. Guan *et al.* presented an analytical method for the reliability and the system response updating without using simulations (Guan *et al.*, 2012). This method has been developed based on the inverse first-order reliability computations via Bayesian modeling to reduce estimation uncertainties.

The major conclusion of methods which have been reviewed in the introduction section can be classified into the following categories:

- There is not a tolerance analysis method which can be modified based on the experimental observations by Bayesian inference.
- The conventional methods by expanding the assembly function into a Taylor series so that an inaccurate linear assembly function with the constant coefficients is obtained.
- There is not a unified tolerance analysis method which can evaluate the reliability of the mechanical assemblies with respect to the quality criteria.
- Some of these approaches are relatively time-consuming and computationally intensive method (e.g. Monte Carlo simulations).

To overcome the above limitations of existing works, in this paper, a novel methodology is proposed for tolerance–reliability of mechanical assemblies with dimensional and geometric tolerances for quality control based on Bayesian modeling.

In general, the tolerance analysis as a key analytical tool can be used to estimate the accumulation of component tolerances on the key specifications of the mechanical assembly in the design stage. On the other hand, due to lack of information about the manufacturing and assembly processes in the design stage, the obtained results of the tolerance analysis may be inaccurate and imprecise. To overcome this weakness, a Bayesian-based model is proposed for tolerance analysis which can be improved by updating based on the experimental data which can be collected from inspection of the physical or virtual prototypes or the quality control of products. In the design

Assembly Automation
Volume 39 · Number 5 · 2019 · 769–782

stage where the experimental observations are not available, for constructing the tolerance analysis model, the prior distribution of the component tolerances can be assumed similar to the conventional methods. For accurately estimating the functional tolerance and the respective reliability evaluation, the uncertainty effects due to the manufacturing and assembly processes can be incorporated into the tolerance analysis model, especially for re-design the product in the mass production.

The available models in the literature (*vector loop*, *variational*, *matrix*, *Jacobian*, *torsor,* etc.) can be applied to build the conventional stack-up function which often is defined as the nominal conditions and analyzed by the classical statistical models such as RSS method. In the presented method, for analyzing the Bayesian probabilistic stack-up function, instead of the classical (frequentist) statistics, the Bayesian statistics is used. Consequently, the initial stack-up function should be appropriately described by a probabilistic model for the Bayesian analysis. The available models in the literature can be applied to build an initial stack-up function at the nominal geometry of the assembly in the first step of the proposed approach. Then, it can be improved by the proposed Bayesian model updating approach based on the posterior experimental observations. In other words, in lack of experimental data for the initial Bayesian regression, the presented method can be used as a complementing tool, not as an alternative to the existing methods in literature, for improving the conventional stack-up function involving the dimensional and geometrical tolerances which can be obtained from the available models in the literature. On the other hand, Monte Carlo method as a data simulation-based approach is the only method in the literature that can be used for both the tolerance and reliability analysis of mechanical assemblies based on the observations. Consequently, it can be an appropriate method for comparing with the proposed Bayesian tolerance – reliability analysis method.

In this paper, unlike to previous methods, a novel integrated tolerance – reliability analysis method is presented for estimating the assembly tolerance and evaluating the reliability of the mechanical assembly with dimensional and geometric tolerances based on the Bayesian modeling.

This paper is organized as follows; In Section 2, the main steps of the proposed method of tolerance Bayesian-based tolerance-reliability analysis are introduced and described in details. Finally, a case study is presented in Section 3 followed by the conclusion in Section 4.

2. Proposed method for tolerance–reliability analysis of mechanical assemblies for quality control based on Bayesian modeling

In this section, a new method for the tolerance-reliability analysis of mechanical assemblies with dimensional and geometric tolerances to control the product's quality is introduced. The proposed method can be carried out in the following stages:

Step 1: Probabilistic modeling of dimensional and geometric tolerances

Step 2: Tolerance analysis based on the Bayesian probabilistic – tolerance model

Step 3: FORM-based reliability analysis for the quality assessment

The proposed steps will be detailed in Sections 2-1, 2-2 and 2-3 respectively.

2.1 Step 1: Probabilistic modeling of dimensional and geometric tolerances

In the design stage, the allowance dimensional and geometrical deviations of individual parts should be limited by the tolerance design procedure. According to the allocated tolerances (design tolerances), the process planning and the manufacturing processes can be conducted in the product development procedure. To extract the assembly function, tolerances should be modeled based on the geometric dimensioning and tolerancing (GD&T) standards (e.g. ASME Y14.5M-2009) (Bendat and Piersol, 2011). The statistical distributions usually indicate the geometrical and dimensional variations due to production processes. Therefore, for tolerance modeling based on the experimental observations from the quality monitoring unit, the probability distribution functions as the probabilistic models can be used.

In the quality control of mechanical assemblies, the random variables, under the normality assumption, can be appropriately described by μ_x and μ_t, as means of an effective variables (x) and the corresponding tolerances (t) respectively, and the same standard deviation (σ). Consequently, in general, the probability distributions of the component dimension (x) and the corresponding tolerance (t) can be expressed as:

$$P_x(x) = N(\mu_x, \sigma) \tag{1}$$

$$P_t(t) = N(\mu_t, \sigma) \tag{2}$$

where $P_x(x)$ and $P_t(t)$ indicate the probability density functions of the component dimension (x) and the corresponding tolerance (t), respectively.

Since the proposed Bayesian – tolerance model is updated based on the obtained experimental data from the quality control unit, the capability of the proposed method for the tolerance analysis of mechanical assemblies with dimensional and geometric tolerances can be directly dependent on the capability of inspecting the geometrical tolerances. In the proposed methodology, all types of tolerances (dimensional and geometrical tolerances) can be handled in the same manner based on the tolerance zone concept. According to GD&T standards (ASME Y14.5M-2009), a tolerance zone is a domain that confines the deviations of a toleranced feature (Standard, 2009, Khodaygan *et al.*, 2010). The tolerance zone usually is a regular zone formed by two parallel straight lines or curves, circles, two concentric circles, two parallel planes or surfaces, parallelepiped faces, cylindrical faces, two coaxial cylinders, or spherical faces (Khodaygan *et al.*, 2010). The size of the tolerance zone depends on tolerance limits, material condition modifiers, and other GD&T rules (Khodaygan *et al.*, 2010, Standard, 2009). To develop the Bayesian – tolerance model based on the proposed algorithm, the dimensional tolerances should be randomly varied according to the corresponding tolerance limits under the specific probabilistic distributions or the experimental observations. For incorporating the geometrical tolerances into the proposed tolerance–reliability analysis method, the size of the tolerance zones can be correspondingly considered as the tolerance limits.

2.2 Step 2: Tolerance analysis based on the Bayesian probabilistic – tolerance model

The dimensions in the mechanical assembly can be classified into two main categories; the manufacturing or component dimensions and the assembly dimensions or key characteristics (KC). The tolerances of the component dimensions are accumulated and directly affect the assembly dimensions. The dimensional and geometrical deviations of the component dimensions can directly affect the functional or assembly dimensions of a mechanical system.

In the proposed method, the tolerance-reliability analysis is carried out based on a probabilistic stack-up model which is called the Bayesian probabilistic – tolerance model. According to the proposed method, first, an initial regression model of the stack-up function is needed. Then, according to the posterior experimental observations, it can be improved based on a Bayesian model updating approach. In general, the Bayesian probabilistic – tolerance model can be expressed as follows:

$$t_y = \theta_1 t_{x_1} + \theta_2 t_{x_2} + \ldots + \theta_n t_{x_n} + \varepsilon \quad (3)$$

where $t_{x_i}$and t_y indicate the tolerances of the independent dimension i and the assembly tolerance of the mechanical assembly, respectively. Also, θ denotes the model parameters and ε is the model error. n and m are the number of parameters and the number of experimental observations, respectively.

In the conventional methods, the parameters of the regression model (θ) are described as the constant coefficients. But in the Bayesian method, the parameters of the regression model are considered as the probabilistic parameters (Wakefield, 2013).

In the proposed method, to find the initial parameters (θ) of the Bayesian probabilistic stack-up model, an initial stack-up function can be built through one of the following approaches;

2.2.1 Fitting a regression model on the prior experimental data

In some applications (especially in the redesign or the reverse design applications), the prior experimental data, which can be collected from inspection of the physical or virtual prototypes or the quality control of the product, may be available for constructing an initial stack-up function. In such cases, a linear regression model can be fitted on the tolerances of the design and the functional or assembly variables based on the available prior experimental data. Consequently, the obtained model can be used as the initial stack-up function for the Bayesian model updating.

2.2.2 Using an available stack-up function

In general, the dependent (functional or assembly) variable of a mechanical assembly (y) can be expressed as a function of the independent (design) variables (x_i, i = 1,. . ., n). This function is usually called the assembly function which can be described in the general explicit form as follows:

$$y = f(x_1,\ x_2, \ldots, x_n) \quad (4)$$

Usually, the stack-up function can be expressed in a linear form by linearizing the assembly function by applying linearization methods based on the sensitivities coefficients.

In some applications, the stack-up function of the mechanical systems can be written in an explicit analytic form. In some complex nonlinear assemblies, deriving an explicit the stack-up function is difficult or impossible.

In the literature, the several efficient methods (*vector loop*, *variational*, *matrix*, *Jacobian*, *torsor*, *parametric*, etc.) have been proposed to build the stack-up function for the tolerance analysis of mechanical assemblies. The available models in the literature can be applied to build an initial linearized stack-up function at the nominal geometry of the assembly in the first step of the proposed approach. Then, it can be improved by the proposed Bayesian model updating approach based on the posterior experimental observations.

2.2.3 Estimating the model parameters (θ) by sensitivity coefficients

Since constructing a proper stack-up function is often difficult or impossible, the initial Bayesian – tolerance model can be built by using the sensitivity coefficients at the nominal geometry within a linear model. In some applications, numerically estimating the sensitivity coefficients is not difficult through the CAD-based parametric models. The sensitivity factors can be estimated by imposing variations into the effective dimensions in the CAD model of the mechanical assembly and then, evaluating the resulted variation in assembly dimension.

As a conclusion, the proposed method for tolerance – reliability analysis needs an initial stack-up model as the prior Bayesian – tolerance model which can be obtained from one of three approaches mentioned above. The posterior probabilistic – tolerance model can be obtained by the Bayesian updating the initial stack-up function based on the posterior experimental observations.

With the assumption of the availability of experimental data, the Bayesian – tolerance model can be expressed as follows:

$$\tilde{t}_y = \tilde{T}\tilde{\theta} + \tilde{\varepsilon} \quad (5)$$

where:

$$\tilde{t}_y = [t_{y_1}, t_{y_2}, \ldots, t_{y_m}]^t$$

$$\tilde{T} = \begin{bmatrix} t_{x_{11}} & \cdots & t_{x_{1k}} \\ \vdots & \ddots & \vdots \\ t_{x_{m1}} & \cdots & t_{x_{mn}} \end{bmatrix}$$

$$\tilde{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]^t$$

$$\tilde{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m]^t$$

where n is the number of parameters and m is the number of experimental observations. $\tilde{t}_y$ and $\tilde{T}$ are the assembly tolerance vector and the effective tolerance matrix, respectively. $\tilde{\theta}$ and $\tilde{\varepsilon}$ indicate the model parameter and model error vectors, respectively.

To determine the unknown parameters θ_i, the least squares method (LSM) can be used (Wakefield, 2013). Accordingly, the sum of squares of the errors ($|\tilde{\varepsilon}|^2$) should be minimized as follows;

$$\tilde{\theta} = \arg\{\min(|\tilde{\varepsilon}^2|)\} \tag{6}$$

where:

$$|\tilde{\varepsilon}^2| = \varepsilon_1^2 + \varepsilon_2^2 + \ldots + \varepsilon_m^2$$

$$\tilde{\theta} = \arg\left\{\min\left(|\tilde{t}_y - \tilde{T}\tilde{\theta}^2|\right)\right\}$$

For minimization of the sum of squares of the errors ($|\tilde{\varepsilon}|^2$), the corresponding partial derivatives as the first-order necessary condition for finding minimum should be equal to zero as follows:

$$\frac{|\partial\tilde{\varepsilon}^2|}{|\partial\tilde{\theta}|} = \frac{|\partial\tilde{t}_y - \tilde{T}\tilde{\theta}^2|}{|\partial\tilde{\theta}|} = 0 \tag{7}$$

From solving equation 7, the coefficients $\tilde{\theta}$ are obtained as:

$$\tilde{\theta} = \left(\tilde{T}^t\ \tilde{T}\right)^{-1}\ \tilde{T}^t\ \tilde{t}_y \tag{8}$$

By assuming that the error terms (ε) are normally distributed with a mean zero ($\varepsilon \sim N\ (0,\sigma)$) and the expected value of the standard deviation s ($E[\sigma]\ =\ s$), the estimated standard deviation ($\tilde{s}$) can be expressed as follows:

$$\tilde{s} = \frac{1}{n-k}\ \tilde{\varepsilon}^t\ \tilde{\varepsilon} \tag{9}$$

By substituting $\tilde{\varepsilon}$ from equation 5 into equation 8, $\tilde{s}$ can be rewritten as follows:

$$\tilde{s} = \frac{1}{n-k}\ \left(\ \tilde{t}_y - \tilde{T}\tilde{\theta}\ \right)^t \left(\ \tilde{t}_y - \tilde{T}\tilde{\theta}\ \right) \tag{10}$$

where the probability density function (PDF) of the computed parameter vector $\tilde{\theta}$ can be assumed t - distribution as follows (Bendat and Piersol, 2011):

$$f(\tilde{\theta}) = \frac{\Gamma[0.5(\nu+k)]s^{-k}\sqrt{\tilde{T}^t\tilde{T}}}{\Gamma[0.5]^k\ \Gamma[0.5\ \ \nu]\nu^{\frac{k}{2}}} \tag{11}$$

where ν indicates the degrees of freedom ($\nu = n - k$).

Accordingly, the probability density function of the variance σ^2 is a chi-square distribution (Bendat and Piersol, 2011):

$$f(\sigma^2) = \nu\ s^2\chi_v^2 \tag{12}$$

Consequently, the tolerance of the assembly variables or the key characteristics (KCs) based on the experimental observation can be estimated via equation (5).

In statistics, there are two main views; classical (frequentist) and Bayesian statistics (Wakefield, 2013, Koch, 2007). In classical statistics, the quantities used to describe the random parameters are unknown constants. In contrast, in Bayesian statistics, parameters are considered as unobserved realizations of random parameters. However, classical and Bayesian statistics differ in their definition of probability. In classical statistics, the probability is the relative frequency of observed data, whereas, in Bayesian statistics, the probability is used to describe parameter's uncertainty without the need for hypothetical replication of the data. In general form, Bayesian inference can be expressed as:

$$\textit{Posterior distribution} \propto \textit{Likelihood function} \times \textit{Prior distribution}$$

Therefore, the posterior distribution is estimated based on the likelihood function and the prior distribution which can be known or assumed about the parameter before the data are collected or analyzed.

Therefore, according to this concept, the Bayesian – tolerance model can be first developed based on a specific assumption about the likelihood function and the prior distribution functions of the parameter vector $(\tilde{\theta})$, in the design stage. Then, based on the collected experimental data from the quality control unit, the Bayesian – tolerance model can be improved by the Bayesian updating procedure for accurately estimating the functional tolerance and the respective reliability evaluation.

Let $\theta\ =\ [\theta_1,\ \ldots,\ \theta_n]^t$ denote all of the unknowns of the model, which we continue to refer to as parameters, and t = $[t_1,\ \ldots,\ t_m]^t$ the vector of observed data. Bayesian inference is based on the posterior probability distribution of θ after observing t, which is given by Bayes theorem (Koch, 2007, Wakefield, 2013):

$$p''(\theta/t) = \frac{l(t/\theta)}{C(t)}p'(\theta) \tag{13}$$

where p' and p'' are the prior and posterior probability, respectively. $l(t/\theta)$ is the likelihood to indicate the compatibility of the evidence t with the given θ. $C(t/\theta)$ is the marginal likelihood that is the same for all possible θ being considered. two key ingredients: the likelihood function $l(t/\theta)$ and the prior distribution $p'(\theta)$.The latter represents the probability beliefs for θ held before observing the data t. The normalizing constant is:

$$c(y)\ =\ \int l(t/\theta)p'(\theta)\ \mathrm{d}\theta \tag{14}$$

and is the marginal probability of the observed data given the model, that is, the likelihood and the prior.

The likelihood function ($l(t/\theta)$ in equation 13) provides the distribution of the data (t) under the given parameter (θ). Consequently, it may be generally several types of likelihood functions such as the binomial likelihood, the normal likelihood, the log-normal likelihood, etc. (Koch, 2007). In the proposed method, the tolerances as the random variables are considered under the normality assumption. Therefore, the normal or Gaussian likelihood as a standard likelihood function can be properly chosen in the proposed tolerance-reliability analysis method.

Assembly Automation
Volume 39 · Number 5 · 2019 · 769–782

The prior distribution ($p'(\theta)$ in equation 13) describes what is known about θ before the experiment data. In general, the prior distribution is "subjective", so its choice is depended on the designer's decision. In the proposed method, the prior distribution is chosen based on the conjugate prior concept. In Bayesian probability theory, if the posterior distribution ($p''(\theta/t)$) is in the same type as the prior probability distribution ($p'(\theta)$), the prior and posterior are then called conjugate distributions, and the prior is called the conjugate prior for the likelihood function (Koch, 2007). In the proposed method, due to the normality assumption of tolerances, the likelihood function can be normal and choose a normal prior distribution can lead to ensuring that the posterior distribution is also normal. Therefore, the normal distribution can be a proper choice for the prior distribution in the proposed method.

The distribution parameters $\theta(\theta \sim N(\mu_\theta, \sigma_\theta))$ themselves randomly are varied according to distribution function ($\mu_\theta \sim N(\mu_0, \sigma_0)$). Based on the experimental observations, new θ is obtained by updating the old θ distribution using the prior conjugate law. According to the prior conjugate law of the normal distribution functions with a constant standard deviation, the posterior mean and standard deviation of μ_θ can be obtained by:

$$\mu''_0 = \frac{-\theta\ \sigma'^2_\theta + \mu'_0\left(\frac{\sigma^2_\theta}{n}\right)}{\sigma'^2_\theta + \frac{\sigma^2_\theta}{n}} \tag{15}$$

$$\sigma''_0 = \left(\frac{\sigma'^2_\theta + \left(\frac{\sigma^2_\theta}{n}\right)}{\sigma'^2_\theta + \frac{\sigma^2_\theta}{n}}\right)^{\frac{1}{2}} \tag{16}$$

where θ and $-\theta$ indicate the random parameter with the normal distribution and its average under n experimental observations, respectively, $'$ and $''$ indicate the prior and posterior, respectively.

2.3 Step 3: FORM-based reliability analysis for the assembly quality assessment

In this step, based on the estimated tolerance of the key characteristics ($\tilde{t}_y$), the reliability of the product to meet the quality requirement(s) is accurately determined.

In general, the reliability analysis can be carried out at two levels: component and system levels. The reliability analysis on component level refers to the probability of satisfying a performance criterion in an individual component of a mechanical system. The reliability analysis at the system level refers to the probability of satisfying a performance criterion at a whole of the system.

Uncertainties due to dimensional and geometrical of tolerances of components can impact on the tolerance of functional characteristics and the reliability performance of mechanical systems. To the quality control of the assembly based on the reliability assessment, two main parameters can be considered; the tolerance of assembly characteristics (t_Y) such as the assembly tolerance and the quality requirement (t_R). To evaluate that the product such that it satisfies the quality requirement towards key characteristics variations, the LSF of quality control can be defined as follows:

$$g(t) = t_R - t_Y(t) \tag{17}$$

where $t_Y(t)$ is a function of the n independent variables $t_1, t_2,. t_n$.

In generally, the LSF can have three outcomes as follows (Figure 1):

$$g(t):\begin{cases} > 0 & \text{Safe region} \\ = 0 & \text{Limit state} \\ < 0 & \text{Failure region} \end{cases}$$

In the tolerance analysis procedure, the tolerance assembly (t_Y) as a probabilistic characteristic can be estimated based on the component tolerances in the mechanical assembly. To control the assembly quality based on the reliability analysis, the assembly tolerance (t_Y) and the tolerance requirement (t_R) can be evaluated via the LSF. The assembly tolerance (t_Y) is considered as a probabilistic characteristic which can be estimated from the proposed Bayesian-based tolerance analysis model. On the other hand, the quality requirement (t_R) can be considered as a deterministic limit or a probabilistic criterion. Failure probability (p_R) for estimating the reliability of the mechanical assembly with respect to the tolerance requirement (t_R) as a ($R = 1 - p_R$) deterministic limit and a probabilistic criterion are shown in Figure 2(a) and (b), respectively.

For reliability analysis of mechanical assemblies, the first-order reliability method (FORM) with low computations can be applied appropriately (Hasofer and Lind, 1974). Based on the FORM, the random variables with different distributions can be transformed into the same standard normal by Rosenblatt transformation and then performs a first-order Taylor expansion at the MPP which has the maximum failure probability on the LSF (Der Kiureghian, 2005).

To estimate the failure probability (p_f) as the probability that the assembly quality or the limit stated may be violated, the following multi-dimensional integral over the failure domain of the performance function ($g < 0$) should be computed:

$$p_f = p(g < 0) = \int_{g<0} \cdots \int f(\tilde{t}_{x_1}, \cdots, \tilde{t}_{x_n})\, d\tilde{t}_{x_1} \cdots d\tilde{t}_{x_n} \tag{18}$$

Where $f(\cdot)$ is the probability density function and the number of integrals (n) is the number of component dimensions as the random variables. According to equation 15, the corresponding integral is calculated in the region that is not desirable for quality control ($g < 0$). The R ($R = 1 - p_f$)value represents the

Figure 1 Several conditions of the LSF

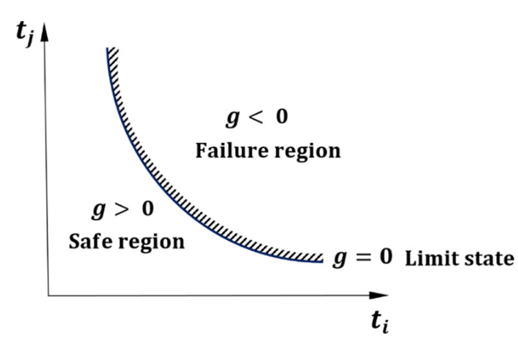
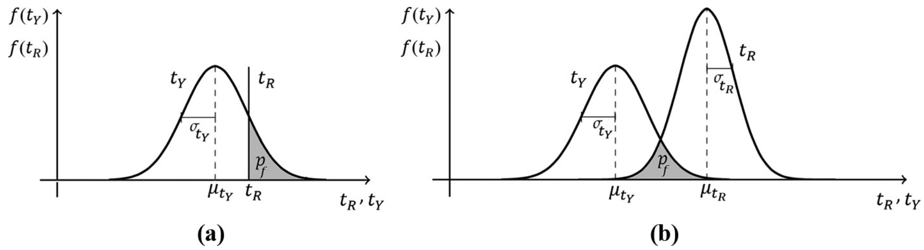


Figure 2 Failure probability (p_R) and the reliability ($R = 1 - p_R$) of the mechanical assembly with considering the assembly tolerance (t_Y) as the probabilistic variable and the tolerance requirement (t_R) as (a) a deterministic threshold (b) a probabilistic criterion



reliability of the product in satisfying the assembly requirement(s).

To solve this problem, the normal standard space is more desirable; therefore the function G as the LSF in the normal standard space instead of g can be used. To transfer the space of the problem into the normal standard space, the conversion of NATAF transformation (Li *et al.*, 2008, Da-gang, 2007) can be used;

$$t_{z_i} = \Phi^{-1}\left(F_i(t_{x_i})\right) \tag{19}$$

where $t_{x_i}$and $t_{z_i}$are the random variable and the random variable, respectively. $F_i(\cdot)$ and $\Phi^{-1}(\cdot)$ indicate the corresponding cumulative distribution function (CDF) of the random variable and the inverse cumulative distribution function of the standard normal variable, respectively. Based on the CDF (Φ) concept, the probability of failure (p_f) can be estimated as follows:

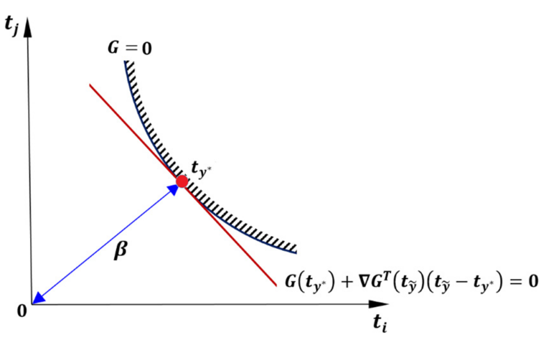
$$p_f = \Phi(-\beta) \tag{20}$$

Where β is called reliability index that indicts (Arora, 2004). The concept of the reliability index (β) with respect to the LSF is visually illustrated in Figure 3. The reliability index (β) can be written as follows:

$$\beta = \frac{\mu_G}{\sigma_G} \tag{21}$$

μ_G and σ_G are the mean and standard deviation of the LSF in the standard normal space. The obtained mean and variance

Figure 3 The concept of the reliability index (β) with respect to the LSF in the standard normal space



from the first term of Taylor series expansion of the LSF can be expressed as follows:

$$\mu_G \approx G\left(\mu_{t_{x_1}}, \mu_{t_{x_2}}, \ldots, \mu_{t_{x_n}}\right) \tag{22}$$

$$\sigma_G^2 \approx \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{n} \frac{\partial G}{\partial t_{x_i}} \frac{\partial G}{\partial t_{x_j}} cov\left(t_{x_i}, t_{x_j}\right) \tag{23}$$

where $Cov(t_{x_i}, t_{x_j})$is the covariance of $t_{x_i}$and t_{x_j}.

For the FORM-based reliability analysis, the LSF can be described by a linear approximation at the design point (t_{y^*}) (Der Kiureghian, 2005).

$$G(t_{\tilde{y}}) \approx G(t_{y^*}) + \nabla G^T(t_{\tilde{y}})(t_{\tilde{y}} - t_{y^*}) \tag{24}$$

The design point (t_{y^*}) is the closest point on $G(t_{\tilde{y}}) \approx 0$ as:

$$t_{y^*} = min\{t_{\tilde{y}} \;\; G(t_{\tilde{y}}) = 0\} \tag{25}$$

where $G(t_{\tilde{y}})$ is the limit-state function in the standard normal space.

Based on solving equation 25, the reliability index β as the distance from the origin in the standard normal space to the point t_{y^*} is calculated as follows (Der Kiureghian, 2005):

$$\beta = |t_{y^*}| \tag{26}$$

To find the design point, the improved Hasofer-Lind-Rackwitz-Fiessler (iHLRF) can be used (Hasofer and Lind, 1974). According to the LSF equation (17) which can be considered at the arbitrary point ($t_{\tilde{y}_m}$) as a candidate of the design point in the iteration m. The new candidate design point ($t_{\tilde{y}_{m+1}}$) can be obtained as follows:

$$t_{\tilde{y}_{m+1}} = t_{\tilde{y}_m} + \delta_m.\ d_m \tag{27}$$

where m is the iteration counter, δ_m and d_m are the step search and the search direction at the m th iteration, respectively.

The proper search direction (d_m) and the step search (δ_m) for carrying out the search algorithms to find the design point are presented as follows (Der Kiureghian, 2005):

Assembly Automation
Volume 39 · Number 5 · 2019 · 769–782

$$d_m = t_{\tilde{y}_{m+1}} - t_{\tilde{y}_m} = \left(\frac{G(t_{\tilde{y}_m})}{||G(t_{\tilde{y}_m})||} + \tilde{\alpha}^T t_{\tilde{y}_m} \right) \tilde{\alpha} - t_{\tilde{y}_m} \tag{28}$$

where can be determined as:

$$\tilde{\alpha} = \frac{\nabla G(t_{\tilde{y}_m})}{||\nabla G(t_{\tilde{y}_m})||} \tag{29}$$

The step size δ_m in equation (27) can be unity by default. According to Armijo rule (Arora, 2004), the step size s_m can be determined as:

$$\delta_m = a^k \tag{30}$$

where a is a positive constant (usually $a = 1/2$) and k is an integer that can be iteratively increased from zero.

It is important to note that convergence criteria to stop the search algorithm for finding the design point can be expressed as follows:

- The design point must be close enough to the surface of the limit state ($G = 0$):

$$\frac{G(\tilde{y}_{m+1}) - G(\tilde{y}_m)}{G(\tilde{y}_m)} \leq e_1 \tag{31}$$

where G is the LSF, and e_1 indicates the corresponding convergence parameter of the stopping criterion that can be taken 0.001.

- The gradient of the surface of the limit state is passed from the source at the last point, which indicates that the current point is the closest point to the origin:

$$\left|\left| \frac{t_{\tilde{y}_m}}{||t_{\tilde{y}_m}||} - \left(\tilde{\alpha}_m \cdot \frac{t_{\tilde{y}_m}}{||t_{\tilde{y}_m}||} \right) \tilde{\alpha}_m \leq e_2 \right|\right| \tag{32}$$

where e_2 is the corresponding convergence parameter of the stopping criterion, that can be selected 0.001.

After finding the design point (t_{y^*}), by considering the equations (18)-(21) at the obtained design point for determining the probability of failure value (p_f), the reliability of the product to satisfy the quality requirement can be estimated as follows:

$$R = 1 - p_f \tag{33}$$

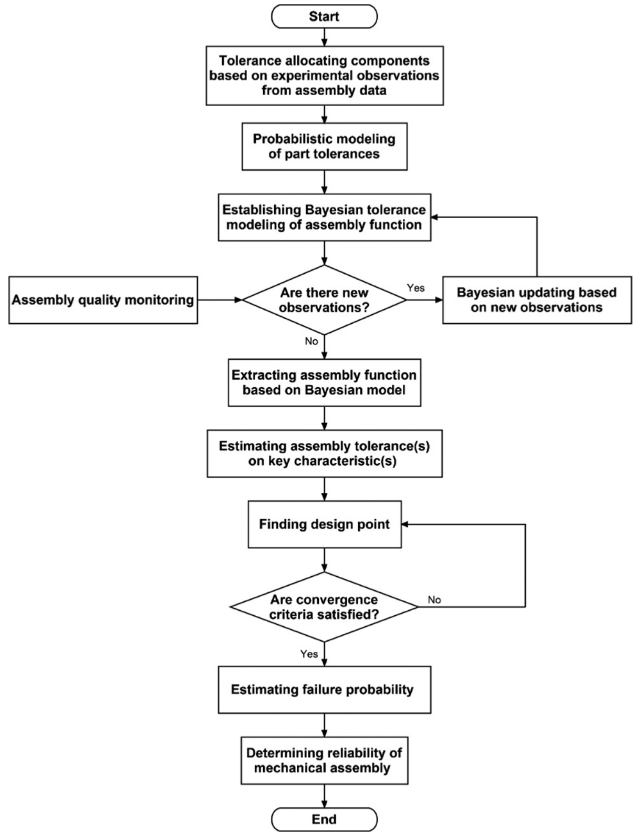
Where R is the reliability of the mechanical assembly to meet the quality assembly requirement. Based on the computed reliability value (R), the probability of satisfying the assembly quality criteria under the tolerances of components are specified.

Finally, for more clarification, the flowchart of the proposed method for the tolerance analysis of the mechanical assembly for quality control based on Bayesian-reliability model is presented (see Figure 4).

For implementing the proposed method within CAD/CAM, the following facilities should be provided:

1 A CAD-based interactive environment within a proper CAD/CAM software (such as SolidWorks®), for parametrically designing the mechanical assembly.

Figure 4 The flowchart of the proposed method



2 A programing environment within the computational software to carry out the following stages:
  - Developing the Bayesian – tolerance model and the Bayesian – updating it based on the proposed algorithm.
  - Evaluating the reliability analysis of the mechanical assembly for the quality assessment established upon the developed code of the proposed FORM-based algorithm.

3 An interface tool to integrate the interactive and computational environments.

To implement the proposed algorithm within the integrated environment, Visual Basic Application (VBA), can be used as a development language integrated tool in a proper CAD/CAM software (such as SolidWorks ®). On the other hand, API (Application Programing Interface) as a software development tool can be used to integrate the different applications. Also, SolidWorks API that covers all the functions of the software can be used to parametrically modify the part model according to user inputs or data from a database (such as generated random values of effective variables). Therefore, to automate the proposed method to use within SolidWorks® as a proper CAD/CAM software, the proposed algorithm can be programed as a VBA code which can be interactively supported for the parametric design of the mechanical assembly using the API functions of SolidWorks®.

Assembly Automation
Volume 39 · Number 5 · 2019 · 769–782

3. Case study

In this section, to illustrate the capability of the proposed method, a one-way clutch with the dimensional and geometric tolerances is investigated as a case study. The components of the clutch assembly and its effective dimensions, and the pressure angle (Y) as its assembly variable are shown in Figure 5. The clutch assembly consists of four components: the hub, the cage, four rollers, and four springs. The one-way clutch assembly is designed to allow rotation in only one direction. According to Figure 5, the assembly function, angle Y, can be expressed in terms of the effective dimensions (i.e. X_1, X_2, X_3 and X_4). The details of dimensions and the corresponding dimensional and geometric tolerances of the one-way clutch assembly are shown in Figure 6. Also, the nominal values of the effective dimensions and the dimensional and geometric tolerances are reported in Table1.

For the tolerance-reliability analysis of the clutch assembly, the proposed method is carried out as follows.

Step 1: Probabilistic modeling of the dimensional and geometric tolerances.

In the first step, the dimensional and geometric tolerances of the clutch assembly are probabilistically modeled. In this study, for modeling the dimensional and geometric tolerances of the clutch assembly, the independence rule, according to ISO 8015 standard, is considered. In this work, the corresponding dimensional and geometric tolerances (according to Figure 5 and Table I) based on the experimental synthetic observations are modeled in the normal distributions. The obtained normal distribution functions of tolerances can be expressed as follows:

$$\begin{aligned}
&t^{D}_{x_1}: N(0.12,\ 0.070)\\
&t^{D}_{x_2}: N(0.05,\ 0.028)\\
&t^{D}_{x_3}: N(0.05,\ 0.026)\\
&t^{D}_{x_4}: N(0.15,\ 0.090)\\
&t^{G}_{x_1}: N(0.05, 0.027)\\
&t^{G}_{x_2}: N(0.02, 0.011)\\
&t^{G}_{x_3}: N(0.02, 0.011)\\
&t^{G}_{x_4}: N(0.07, 0.038)
\end{aligned} \tag{34}$$

where $t^{D}_{x_i}$and $t^{G}_{x_i}$ represents the dimensional and geometric tolerances of the dimension i (x_i), respectively. For examining the normality assumption, the data-based distributions of dimensional and geometric tolerances are compared to the normal distribution function (Figures 7 and 8). Figures 7 and 8 show that the normal assumption is a pretty good approximation for the probabilistic modeling of the dimensional and geometric tolerances. For analysis of covariance, the covariance matrix of tolerances can be obtained as follows:

$$Cov(t_{x_i}, t_{x_j}) = \begin{bmatrix}
1 & -0.0041 & -0.1644 & 0.1428 & 0.0133 & 0.0242 & 0.0055 & 0.0968\\
-0.0041 & 1 & 0.0083 & 0.0956 & -0.0418 & -0.0426 & -0.0782 & -0.0395\\
-0.1644 & 0.0083 & 1 & 0.0854 & -0.0968 & 0.0212 & 0.0418 & -0.0478\\
0.1428 & 0.0956 & 0.0854 & 1 & -0.0399 & -0.0961 & -0.0623 & 0.0737\\
0.0133 & -0.0418 & -0.0968 & -0.0399 & 1 & -0.0052 & 0.0969 & 0.1729\\
0.0242 & -0.0426 & 0.0212 & -0.0961 & -0.0052 & 1 & 0.0099 & -0.0228\\
0.0055 & -0.0782 & 0.0418 & -0.0623 & 0.0969 & 0.0099 & 1 & 0.0218\\
0.0968 & -0.0395 & -0.0478 & 0.0737 & 0.1729 & -0.0228 & 0.0218 & 1
\end{bmatrix} \tag{35}$$

Figure 5 The schematic of the one-way clutch assembly; (a) the components (b) the effective dimensions (X), and the assembly variable (Y)

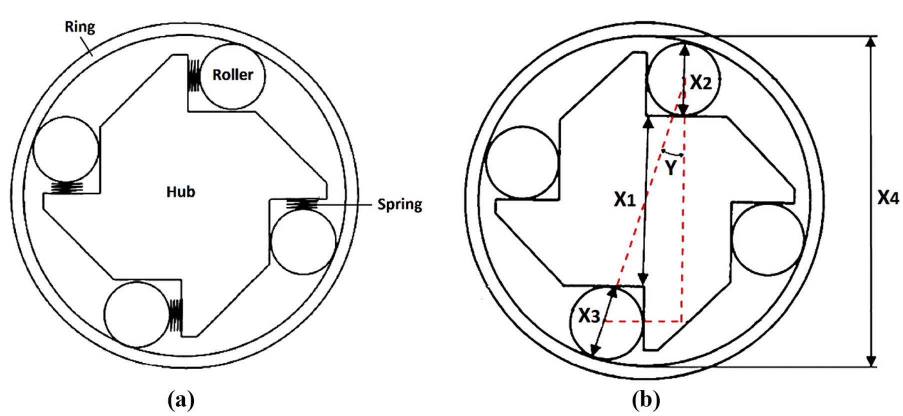
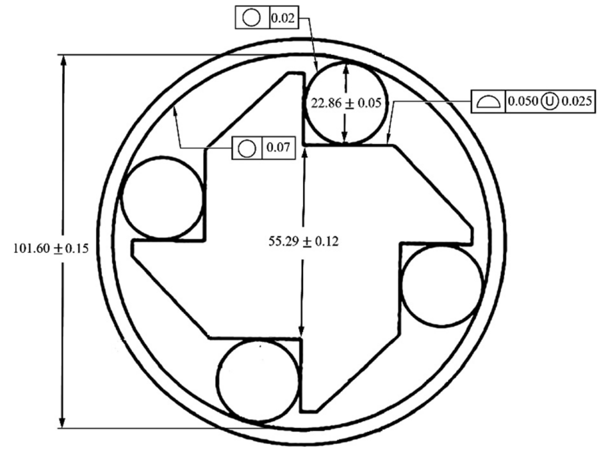


Figure 6 The details of dimensions and the corresponding dimensional and geometric tolerances of one-way clutch assembly



Step 2: Tolerance analysis based on the Bayesian probabilistic – tolerance model.

Based on the proposed method, a Bayesian linear regression model as the Bayesian – tolerance model or the probabilistic stack-up function can be created using the experimental data.

In this study, the obtained Bayesian – tolerance model can be expressed as follows:

$$t_y = \theta_1 t_{x_1}^D + \theta_2 t_{x_2}^D + \theta_3 t_{x_3}^D + \theta_4 t_{x_4}^D + \theta_5 t_{x_1}^G + \theta_6 t_{x_2}^G + \theta_7 t_{x_3}^G + \theta_8 t_{x_4}^G + \theta_9 + \varepsilon \tag{36}$$

where t_y is the tolerance of the pressure angle (the assembly variable) and t_{x_i} indicates the tolerance of the design variable i. θ_i denotes the model parameters and ε is the model error with the normal distribution. The mean and standard deviation of $\widetilde{\theta}$ are computed as follows:

$$E(\tilde{\theta}) = (0.1163,\ 0.1124,\ 0.1176,\ 0.1157,\ 0.1159,\ 0.1150, 0.1007,\ 0.1163, 0.1218)$$

$$\sigma\tilde{\theta} = (0.106,\ 0.260, 0.285, 0.083, 0.027, 0.636, 0.648, 0.194, 0.036) \tag{37}$$

where $\widetilde{\theta} = [\theta_1, \theta_2, \ldots, \theta_9]^t$ and $\sigma_{\widetilde{\theta}} = [\sigma_{\theta_1}, \sigma_{\theta_1}, \ldots, \sigma_{\theta_9}]^t$ are the vector of the model parameters and its standard deviation vector, respectively.

Also, the correlation matrix of coefficients is computed as below:

$$\mathrm{Cov}(\theta_i, \theta_j)\begin{bmatrix} 1 & 0.0120 & 0.1779 & -.1582 & 0.0135 & -0.0453 & -0.0214 & -0.0797 & -0.3202 \\ 0.0120 & 1 & 0.0022 & -0.0900 & 0.0244 & 0.0339 & 0.0685 & 0.0396 & -0.3845 \\ 0.1779 & 0.0022 & 1 & -0.1158 & 0.0914 & -0.0357 & -0.0599 & 0.0239 & -0.4159 \\ -0.1582 & -0.0900 & -0.1158 & 1 & -0.0705 & 0.0064 & 0.1797 & 0.0864 & -0.3685 \\ 0.0135 & 0.0244 & 0.0914 & -0.0705 & 1 & -0.0108 & -0.0762 & 0.0170 & -0.2754 \\ -0.0453 & 0.0339 & -0.0357 & 0.0064 & -0.0108 & 1 & -0.0783 & 0.0724 & -0.4337 \\ -0.0214 & 0.0685 & -0.0599 & 0.1797 & -0.0762 & -0.0783 & 1 & 0.0442 & -0.3826 \\ -0.0797 & 0.0396 & 0.0239 & 0.0864 & 0.0170 & 0.0724 & 0.0442 & 1 & -0.4592 \\ -0.3202 & -0.3845 & -0.4159 & -0.3658 & -0.2754 & -0.4337 & -0.3826 & -0.4592 & 1 \end{bmatrix} \tag{38}$$

Consequently, the error distribution is obtained as follows:

$$\varepsilon : N(0,\ 0.001) \tag{39}$$

To evaluate the obtained Bayesian tolerance model, the corresponding R-factor and the normality of the model can be examined. The computed R-factor of the Bayesian linear regression model is 0.995, which illustrates a good agreement between the model and the experimental data. Also, Figure 9 shows the obtained model has the good agreement with normality assumption.

Table I Effective dimensions and corresponding specifications of the one-way clutch

Componeznt	Dimension (mm)		Dimensional Tolerances (mm)		Geometric Tolerances (mm)	
Height of hub	x_1	55.29	$t_{x_1}^D$	±0.12	$t_{x_1}^G$	0.05
Upper roller diameter	x_2	22.86	$t_{x_2}^D$	±0.05	$t_{x_2}^G$	0.02
Lower roller diameter	x_3	22.86	$t_{x_3}^D$	±0.05	$t_{x_3}^G$	0.02
Cage diameter	x_4	101.60	$t_{x_4}^D$	±0.15	$t_{x_4}^G$	0.07

Figure 7 Evaluating the normality assumption of the probability distributions of dimensional tolerances

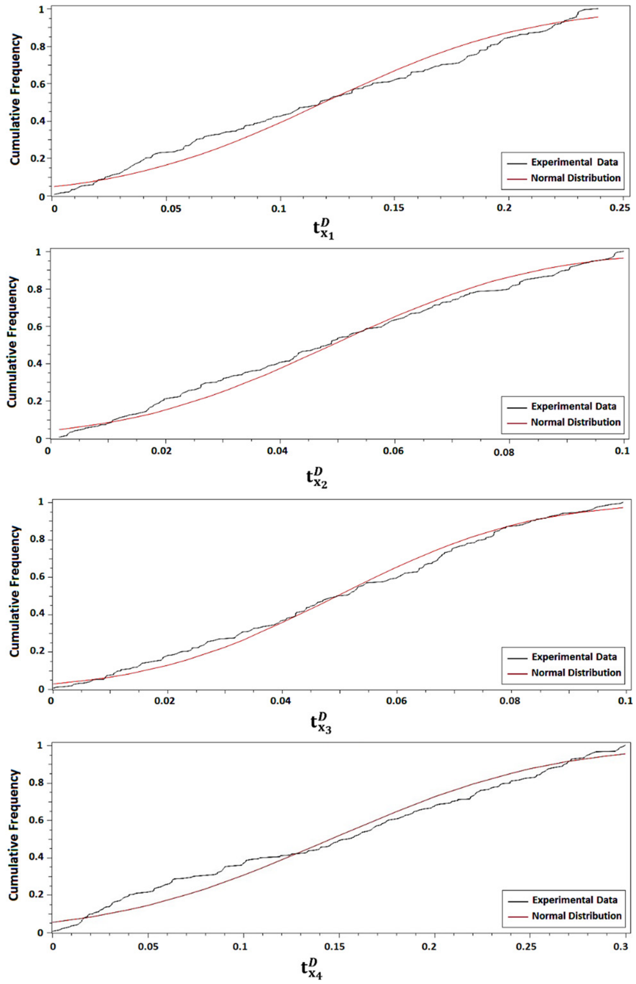
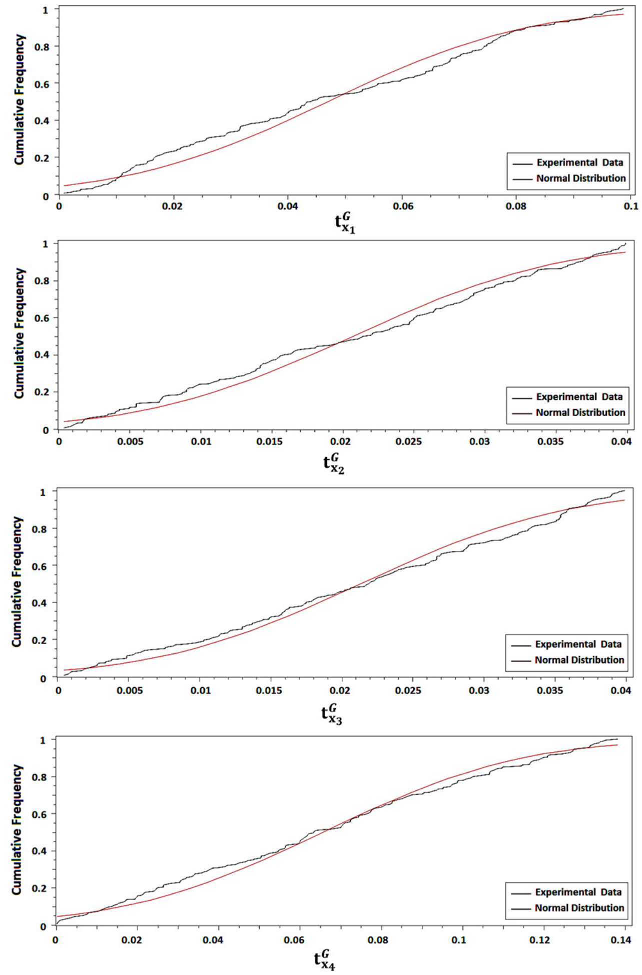


Figure 8 Evaluating the normality assumption of the probability distributions of geometric tolerances



Step 3: FORM-based reliability analysis for the assembly quality assessment

In this step, the assembly reliability for assessing the quality requirement is estimated using the proposed FORM-based method. Therefore, the LSF can be considered as follows according to equation (14)]:

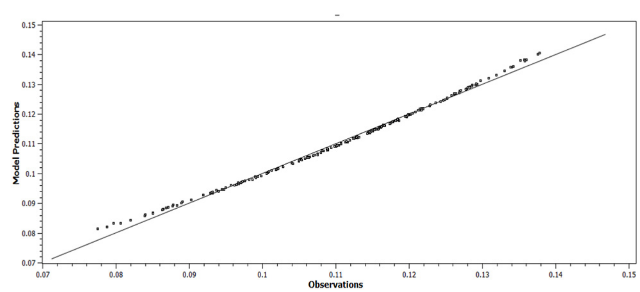
$$g(\tilde{t}_x) = 1.4 - t_Y(\tilde{t}_x) \tag{40}$$

where t_Y indicates the Bayesian linear regression model [equation (30)]. In this study, the quality requirement of the key characteristic [t_R in equation (14)] is 1.4 degrees.

Based on the proposed method, the computed reliability value is 99.61 per cent. Therefore, the probability of satisfying the assembly requirement in the quality control unit is 99.61 per cent. In other words, the probability of violation of the limit state (i.e. the pressure angle 1.4°) in the assembly process is 0.39 per cent (p_f=0.39 per cent).

It should be emphasized that the proposed method consists of two main phases; (1) Bayesian-based tolerance analysis (2) First Order Method (FORM) – based reliability analysis. On the other hand, Monte Carlo simulation is the only method in

Figure 9 Evaluating normality of the obtained Bayesian - tolerance model



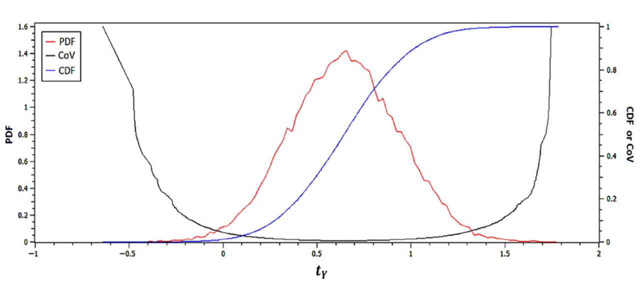
the literature that both the tolerance and reliability analysis can be cried out based on it. Therefore, it can be a proper method for comparing with the proposed method. To verify the obtained results of the assembly tolerance and the reliability analysis from the proposed method, the computational results of the proposed method are compared to the obtained results from the Monte Carlo simulations. Accordingly, the assembly tolerance and the reliability value of the one-way clutch assembly are estimated using 100000 simulations of Monte Carlo method at the same condition. The cumulative distribution function (CDF), the probability density function, and the covariance (CoV) of the Monte Carlo simulations are shown in Figure 10.

The comparing the obtained results of the proposed method and Monte Carlo simulation are reported in Table II. According to Table II, the obtained results of the assembly tolerance (the mean (μ_{t_Y}) and the standard deviation(σ_{t_Y}) and the reliability value from the proposed method, respectively have (0.98 per cent, 2.55 per cent) and 0.1 per cent errors in comparing to the obtained results of Monte Carlo simulation. Therefore, the results of the proposed method under the low computational time are in very good agreement with the accurate results of Monte Carlo simulation approach as a time-consuming and computationally intensive method.

On the other hand, the several methods for tolerances analysis of mechanical assemblies have been proposed in the literature. Hence, the proposed method as a Bayesian statistics-based method is compared with a classical statistics-based method from the literature in the tolerances analysis of a mechanical assembly with the dimensional and geometric tolerances. Therefore, according to this comment, the one-way clutch assembly with dimensional and geometrical tolerances has been considered by the vector-loop method under classical statistical analysis (RSS method). The obtained results from the proposed method, the vector-loop model, and Monte Carlo simulations in terms of the mean value (μ_{t_Y}) and standard deviation (σ_{t_Y}) of the assembly tolerance (t_Y) under the normality assumption, has been reported in Table III. According to Table III, the relative error of the upper limit of the assembly tolerance based on the 3-sigma concept $(UL_{t_Y} = \mu_{t_Y} + 3\sigma_{t_Y})$ from the proposed method and the vector loop method with respect to the Monte Carlo simulations are 2 per cent and 6 per cent, respectively.

As a conclusion, in the tolerance analysis phase, the proposed method in comparing to the vector-loop method (as a classical statistics-based method in the literature) is a more accurate method. In lack of prior experimental data for building the initial stack-up function, the proposed method may have a more computational cost due to the Bayesian model updating procedure which can be added to same computations of constructing the initial stack-up function. But, in conditions that the experimental data is available, the computational cost of building the stack-up function and its analysis based on the Bayesian-based method may be lower than the overall

Figure 10 The cumulative distribution function (CDF), the probability density function (PDF), and the covariance (CoV) of the assembly tolerance (t_Y) based on the Monte Carlo simulations



computations of establishing the vector-loop, conducting the direct linearization method (DLM) and the classical statistical analysis. However, in both the tolerance and reliability analysis phases, the proposed method can be compared to the Monte Carlo simulations method as the only method in the literature that is capable for conducting both the tolerance and reliability analysis. The obtained results show that the proposed method, in the tolerance-reliability analysis, with the low computational time can be present the accurate results in comparing with the results of Monte Carlo simulations approach as an iterative and time-consuming method.

4. Conclusion

The components of a mechanical assembly are inherently involved with dimensional and geometrical errors. The tolerances of components are accumulated on the key characteristics and affect the assemblability, the functionality, and the quality of the mechanical assembly. In this paper, unlike conventional methods, a novel approach was presented for the tolerance–reliability analysis of mechanical assemblies to estimate the tolerance assembly and the reliability of the mechanical assembly to meet the quality requirement(s) at the actual condition through the Bayesian modeling. Using Bayesian modeling, the accurate assembly function of the complex mechanical assembly can be formulated based on the experimental observations. According to the proposed method, first, tolerances are modeled as the random variables based on the experimental observations. Then, the explicit assembly function can be modeled established upon the Bayesian inference regarding manufacturing and assembly tolerances. Consequently, the reliability of the mechanical assembly to satisfy the assembly requirement(s) can be estimated by a proper first order-reliability method. Finally, to demonstrate the efficiency of the proposed method, a one-way clutch was considered as a case study, and the obtained results were compared to the results of the Monte Carlo simulations. The computational results from the proposed method were in a good agreement with the obtained results from the Monte Carlo simulations. The proposed method can be automated to use within CAD/CAM software for utilization in the industrial applications. Major advantages of the proposed method can be summarized as follows:

- To reach an accurate model for tolerance–reliability analysis, a new formulation model has been proposed which its coefficients are not constant and can be modified as the probabilistic parameters by Bayesian updating based on the experimental observations.
- In a unified manner, it can accurately estimate functional tolerance and the assembly reliability of the mechanical assembly.
- The proposed method can be capable of handling the mechanical assemblies with non-linear assembly function and all types of tolerances for the tolerance-reliability analysis.
- The proposed method can yield high accuracy with low computational cost.

However, as a significant limitation of the proposed method, its efficiency is limit to tolerance – reliability analysis of the mechanical assemblies with the low flexibility of components.

Table II Comparing the obtained results from the proposed method and Monte Carlo simulations

Outputs of analysis	Proposed method	Monte Carlo simulations	Relative error
Assembly tolerance (t_Y: $N(\mu_{t_Y}, \sigma_{t_Y})$)	N (0.603°,0.281°)	N (0.609°,0.274°)	(0.98%, 2.55%)
Assembly reliability (R)	99.61%	99.72%	0.1%

Table III The obtained assembly tolerance (t_Y) from the proposed method, the vector-loop method, and Monte Carlo simulations

Assembly tolerance (t_y)	Proposed method	Vector-loop method	Monte Carlo simulations
$N(\mu_{t_Y}, \sigma_{t_Y})$	N (0.603°, 0.281°)	N (0°, 1.327°)	N (0.609°, 0.267°)
$UL_{t_Y} = \mu_{t_Y} + 3\sigma_{t_Y}$	1.446°	1.327°	1.410°

Therefore, it is not proper for analysis of the flexible assemblies with large deformation.

References

Armillotta, A. (2014), "A static analogy for 2D tolerance analysis", *Assembly Automation*, Vol. 34 No. 2, pp. 182-191.

Arora, J.S. (2016), *Introduction to Optimum Design*, 4th Edition, Academic Press, Elsevier, New York, NY.

Bendat, J.S. and Piersol, A.G. (2011), *Random Data: analysis and Measurement Procedures*, John Wiley & Sons.

Bernstein, N. (1989), "Representation of tolerance information in solid models", *Advances in Design Automation, ASME*, pp. 37-48.

Bucher, C.G. and Bourgund, U. (1990), "A fast and efficient response surface approach for structural reliability problems", *Structural Safety*, Vol. 7 No. 1, pp. 57-66.

Chase, K.W., Gao, J. and Magleby, S.P. (1995), "General 2-D tolerance analysis of mechanical assemblies with small kinematic adjustments", *Journal of Design and Manufacturing*, Vol. 5, pp. 263-274.

Chase, K.W. and Parkinson, A.R. (1991), "A survey of research in the application of tolerance analysis to the design of mechanical assemblies", *Research in Engineering Design*, Vol. 3 No. 1, pp. 23-37.

Da-Gang, L. (2007), "First order reliability method based on linearized nataf transformation", *Engineering Mechanics*, Vol. 5, p. 13

Davidson, J., Mujezinovic, A. and Shah, J. (2002), "A new mathematical model for geometric tolerances as applied to round faces", *Journal of Mechanical Design*, Vol. 124 No. 4, pp. 609-622.

der Kiureghian, A. (2005), First-and second-order reliability methods. *Engineering design reliability handbook*, Vol. 14.

Dimov, I.T. (2008), *Monte Carlo Methods for Applied Scientists*, World Scientific.

Du, X. and Hu, Z. (2012), "First order reliability method with truncated random variables", *Journal of Mechanical Design*, Vol. 134 No. 9, p. 91005

Gelman, A. and Meng, X.-L. (1998), "Simulating normalizing constants: from importance sampling to bridge sampling to path sampling", *Statistical Science*, pp. 163-185.

Greenwood, W. and Chase, K. (1988), "Worst case tolerance analysis with nonlinear problems", *Journal of Engineering for Industry*, Vol. 110 No. 3, pp. 232-235.

Greenwood, W. and Chase, K. (1990), "Root sum squares tolerance analysis with nonlinear problems", *Journal of Engineering for Industry*, Vol. 112 No. 4, pp. 382-384.

Guan, X., He, J., Jha, R. and Liu, Y. (2012), "An efficient analytical bayesian method for reliability and system response updating based on laplace and inverse first-order reliability computations", *Reliability Engineering & System Safety*, Vol. 97, pp. 1-13.

Hasofer, A.M. and Lind, N.C. (1974), "Exact and invariant second-moment code format", *Journal of the Engineering Mechanics Division*, Vol. 100, pp. 111-121.

Jayaraman, R. and Srinivasan, V. (1989), "Geometric tolerancing: i Virtual boundary requirements", *IBM Journal of Research and Development*, Vol. 33 No. 2, pp. 90-104.

Khodaygan, S. and Movahhedy, M. (2011), "Tolerance analysis of assemblies with asymmetric tolerances by unified uncertainty–accumulation model based on fuzzy logic", *The International Journal of Advanced Manufacturing Technology*, Vol. 53 Nos 5/8, pp. 777-788.

Khodaygan, S. and Movahhedy, M. (2016), "A comprehensive fuzzy feature-based method for worst case and statistical tolerance analysis", *International Journal of Computer Integrated Manufacturing*, Vol. 29, pp. 42-63.

Khodaygan, S., Movahhedy, M. and Fomani, M.S. (2010), "Tolerance analysis of mechanical assemblies based on modal interval and small degrees of freedom (MI-SDOF) concepts", *The International Journal of Advanced Manufacturing Technology*, Vol. 50 Nos 9/12, pp. 1041-1061.

Koch, K.-R. (2007), *Introduction to Bayesian Statistics*, Springer Science & Business Media.

Laperrière, L. and Lafond, P. (1999), *Tolerance Analysis and Synthesis Using Virtual Joints, Global Consistency of Tolerances*, Springer.

Li, H., Lü, Z. and Yuan, X. (2008), "Nataf transformation based point estimate method", *Science Bulletin*, Vol. 53, p. 2586

Liu, J.S. (1996), "Metropolized independent sampling with comparisons to rejection sampling and importance sampling", *Statistics and Computing*, Vol. 6 No. 2, pp. 113-119.

Madsen, H.O., Krenk, S., and Lind, N.C. (2006), *Methods of Structural Safety*, Courier Corporation.

Pasupathy, T.K., Morse, E.P. and Wilhelm, R.G. (2003), "A survey of mathematical methods for the construction of geometric tolerance zones", *Journal of Computing and Information Science in Engineering*, Vol. 3, pp. 64-75.

Polini, W. (2016), "Concurrent tolerance design", *Research in Engineering Design*, Vol. 27 No. 1, pp. 23-36.

Polini, W. and Corrado, A. (2016), "Geometric tolerance analysis through jacobian model for rigid assemblies with translational deviations", *Assembly Automation*, Vol. 36 No. 1, pp. 72-79.

Rebba, R. and Mahadevan, S. (2008), "Computational methods for model reliability assessment", *Reliability Engineering & System Safety*, Vol. 93, pp. 1197-1207.

Requicha, A.A. (1983), "Toward a theory of geometric tolerancing", *The International Journal of Robotics Research*, Vol. 2 No. 4, pp. 45-60.

Standard, A. (2009), "Dimensioning and tolerancing–engineering drawing and related documentation practices", *Revision of ASME Y*, Vol. 14.

Wade, O. (1983), "Tolerance control", *Tool and Manufacturing Engineers Handbook*, Vol. 1, pp. 2-1.

Wakefield, J. (2013), *Bayesian and Frequentist Regression Methods*, Springer Science & Business Media.

Corresponding author

S. Khodaygan can be contacted at: khodaygan@sharif.edu

For instructions on how to order reprints of this article, please visit our website:
www.emeraldgrouppublishing.com/licensing/reprints.htm
Or contact us for further details: **permissions@emeraldinsight.com**